



TOPIC OVERVIEW

Topic C — Angle Relationships

Triangle Sum, Exterior Angles, and Parallel Lines



This overview guide contains...

The topic calendar — every objective and scheduled assessment across this part of the unit

Vocabulary — the terms you must build into your artifacts

Check Your Understanding problem sets — an extra tool to monitor your progress. Completing all questions is not required, but a guide kept up with, completed in full, and turned in at the end of the quarter raises your Standards Mastery Check score by **5%**. Answer keys for every question are posted on the class website.

Since this guide contains so much foundational information vital to success in this course, it is **essential** to keep up with it. It's THE **most important document** needed for success.

Topic Calendar

Monday, September 14th – Friday, September 18th

ARTIFACT 8 · Triangle Sum Theorem

ARTIFACT 9 · Exterior Angle Theorem

WEEK 6 SEPTEMBER 14–18

BLOCK 1

MON–TUE, SEP 14–15

I can complete a statement that describes an angle relationship when given a partially labeled diagram and three labeled diagrams of an angle relationship. **8.8D**

I can determine missing angle measures when given a partially labeled diagram. **8.8D**

ARTIFACT 10 · Parallel Lines and Transversals

BLOCK 2

WED–THU, SEP 16–17

Block 1 objectives, continued. **8.8D**

Public Presentation · Improve Dallas

preparation introduced Wed–Thu, Sep 16–17 · presentations due Friday, Sep 25

Unit 1 Mid-Unit Assessment · all TEKS to date (8.10A–D, 8.3A–C)

Friday, Sep 18 · revisions due Tuesday, Sep 29 · also this day: Q1 Mathematical Process, Weeks 4–6

AFTER THE TOPIC

Performance Task 1.3 revisions due Tuesday, Sep 22

Performance Task 1.4 · Angle Relationships

in class Wed–Thu, Sep 23–24 · revisions due Wednesday, Oct 7

Vocabulary

All of these terms must be eventually added to at least one artifact (Artifact 8, 9, and/or 10!)

angle	two rays sharing an endpoint (the vertex); measured in degrees
vertex	the common endpoint of the two rays of an angle
degree	the unit for measuring angles; a full turn is 360°
right angle	an angle that measures exactly 90° (shown by a small square)
straight angle	an angle that measures 180° (a straight line)
adjacent angles	two angles that share a vertex and a side but do not overlap
complementary angles	two angles whose measures add to 90°
supplementary angles	two angles whose measures add to 180°
linear pair	two adjacent angles whose outer sides form a straight line; they are supplementary
vertical angles	the opposite angles formed by two intersecting lines; they are equal
congruent angles	angles with equal measure
interior angle	an angle inside a triangle
triangle sum theorem	the three interior angles of any triangle add to 180°
exterior angle	the angle formed outside a triangle when one side is extended
remote interior angles	the two interior angles not next to a given exterior angle
exterior angle theorem	an exterior angle equals the sum of its two remote interior angles
isosceles triangle	a triangle with two equal sides and two equal base angles
parallel lines	lines in the same plane that never meet (marked with arrows)

transversal	a line that crosses two or more other lines
corresponding angles	angles in the same position at each crossing; equal when the lines are parallel
alternate interior angles	angles between the two lines, on opposite sides of the transversal; equal when parallel
same-side interior angles	angles between the two lines, on the same side of the transversal; supplementary when parallel
similar triangles	triangles with equal corresponding angles and proportional sides (symbol $\sim$)
angle-angle (AA) criterion	if two angles of one triangle equal two angles of another, the triangles are similar

Check Your Understanding

Topic C – Angle Relationships

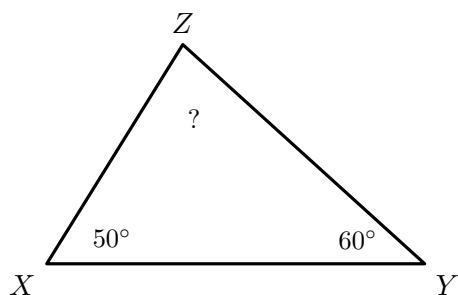
Triangle Sum, Exterior Angles, and Parallel Lines

Questions 1-9 cover Artifact 8 (Triangle Sum Theorem)

1. State the **triangle sum theorem**, and in one sentence explain what it lets you find.

The triangle sum theorem states that all three interior angles of any triangle add to 180° .

2. Find the unknown angle in each triangle. Show the sum you used.



Find the measure of angle Z.

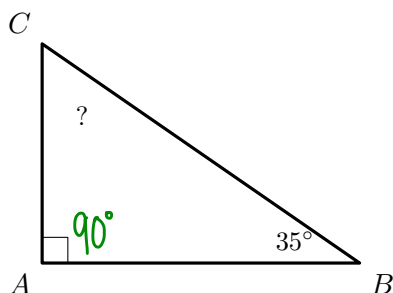
→ 70°

*Triangle Sum Theorem
(all interior angles add to 180°)

$$50 + 60 + x = 180$$

$$\begin{array}{r} 110 + x = 180 \\ -110 \quad -110 \quad \underline{x = 70} \end{array}$$

3. A right triangle is shown.



Find the measure of angle C. State how the right angle helps.

↳ This reveals the second angle measure → 90°

$$90 + 35 + x = 180$$

$$\begin{array}{r} 125 + x = 180 \\ -125 \quad -125 \end{array}$$

$$\underline{x = 55}$$

↓
 55°

4. The interior angles of a triangle measure $(2x)^\circ$, $(3x)^\circ$, and 80° . Write an equation for the angle sum, solve for x , and find all three angle measures.

$$2x + 3x + 80 = 180 \quad \text{*Triangle Sum Theorem}$$

$$\begin{array}{r} 5x + 80 = 180 \\ -80 \quad -80 \\ \hline 5x = 100 \\ \frac{5x}{5} = \frac{100}{5} \\ x = 20 \end{array}$$

$$\begin{array}{l} 2x = 2(20) = 40^\circ \\ 3x = 3(20) = 60^\circ \\ 80^\circ \end{array}$$

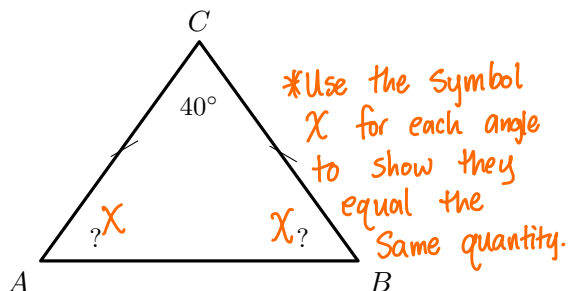
5. The three interior angles of a triangle measure x° , $(x+20)^\circ$, and $(x+40)^\circ$. Write an equation, solve for x , and find each angle.

$$x + x + 20 + x + 40 = 180$$

$$\begin{array}{r} 3x + 60 = 180 \\ -60 \quad -60 \\ \hline 3x = 120 \\ \frac{3x}{3} = \frac{120}{3} \\ x = 40 \end{array}$$

$$\begin{array}{l} x = 40^\circ \\ x + 20 = 40 + 20 = 60^\circ \\ x + 40 = 40 + 40 = 80^\circ \end{array}$$

6. An isosceles triangle has the apex angle and two equal base angles shown.



The two base angles are equal. Find the measure of each base angle. Show your reasoning.

$$x + x + 40 = 180$$

$$\begin{array}{r} 2x + 40 = 180 \\ -40 \quad -40 \\ \hline 2x = 140 \\ \frac{2x}{2} = \frac{140}{2} \\ x = 70 \end{array}$$

Each base angle is 70°

7. In an isosceles triangle, each base angle measures $(2x)^\circ$ and the apex angle measures $(x+20)^\circ$. Write an equation for the angle sum, solve for x , and find all three angles.

$$2x + 2x + x + 20 = 180$$

$$\begin{array}{r} 5x + 20 = 180 \\ -20 \quad -20 \\ \hline 5x = 160 \\ \frac{5x}{5} = \frac{160}{5} \\ x = 32 \end{array}$$

$$\begin{array}{l} 2x = 2 \cdot 32 = 64^\circ \text{ (base } \angle\text{s)} \\ x + 20 = 32 + 20 = 52^\circ \text{ (apex)} \end{array}$$

8. The interior angles of a triangle measure x° , $(2x-10)^\circ$, and $(x+30)^\circ$. Write an equation, solve for x , and find each angle.

$$x + 2x - 10 + x + 30 = 180$$

$$\begin{array}{r} 4x + 20 = 180 \\ -20 \quad -20 \\ \hline 4x = 160 \\ \frac{4x}{4} = \frac{160}{4} \\ x = 40 \end{array}$$

$$\begin{array}{l} x = 40^\circ \\ 2(40) - 10 = 70^\circ \\ 40 + 30 = 70^\circ \end{array}$$

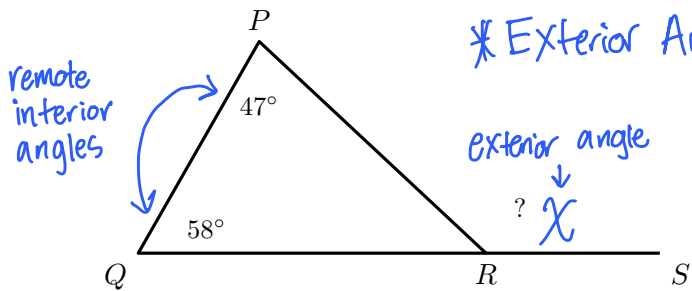
9. A student claims a triangle can have two right angles. Use the triangle sum theorem to explain why this is impossible.

*A triangle must always have 3 angles, which add to 180° (triangle sum theorem)

*If two angles are 90° ($90 + 90$ is already 180°), that leaves no possibility for a third angle $\rightarrow$ forming a triangle is impossible.

Questions 10-18 cover Artifact 9 (Exterior Angle Theorem)

10. Side QR of triangle PQR is extended to S . Find the exterior angle marked ? using only the two remote interior angles, and name the relationship you used.

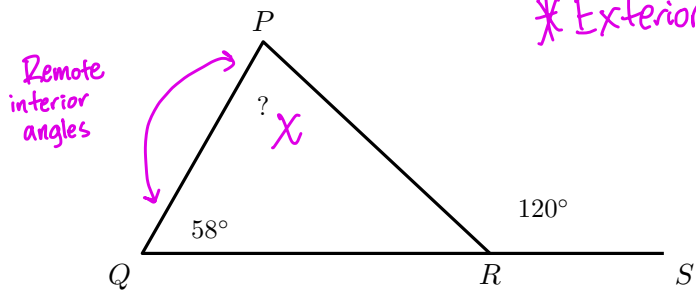


* Exterior Angle Theorem

Find the exterior angle at R .

$$\begin{aligned} x &= 58 + 47 \\ &= \underline{105^\circ} \end{aligned}$$

11. In triangle PQR , side QR is extended to S . The exterior angle at R is 120° and one remote interior angle is 58° .

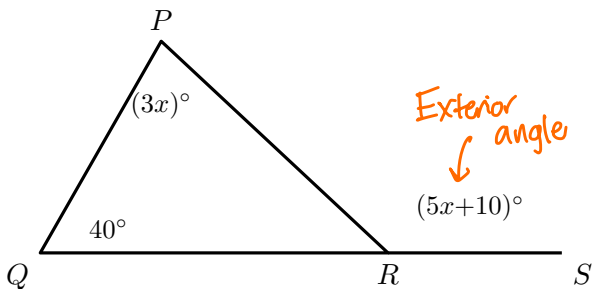


* Exterior Angle Theorem

Find the remote interior angle at P .

$$\begin{aligned} 120 &= 58 + x \\ -58 &-58 \\ \hline 62^\circ &= x \end{aligned}$$

12. An exterior angle of a triangle measures $(5x + 10)^\circ$. Its two remote interior angles measure $(3x)^\circ$ and 40° .



Write an equation using the exterior angle theorem, solve for x , and find the exterior angle.

$$\begin{aligned} 40 + 3x &= 5x + 10 \\ -3x &-3x \\ \hline 40 &= 2x + 10 \\ -10 &-10 \\ \hline 30 &= 2x \\ x &= 15 \end{aligned}$$

exterior angle

$$5(15) + 10 = 85^\circ$$

13. An exterior angle of a triangle measures 140° , and its two remote interior angles measure $(4x)^\circ$ and $(3x)^\circ$. Write an equation, solve for x , and find both remote interior angles.

$$140 = 4x + 3x$$

$$\frac{140}{7} = \frac{7x}{7}$$

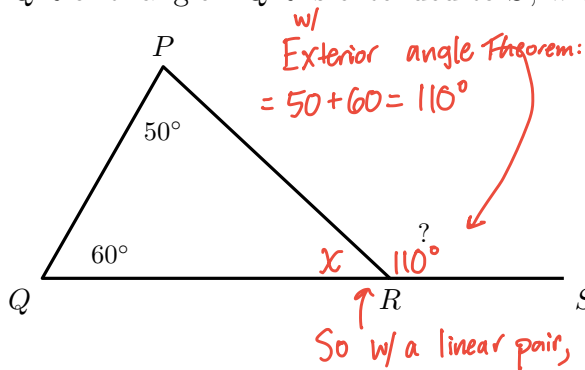
$$\underline{20 = x}$$

$$4(20) = \underline{80^\circ}$$

$$3(20) = \underline{60^\circ}$$

remote interior angles

14. Side QR of triangle PQR is extended to S , with the two remote interior angles shown.



Find the *interior* angle of the triangle at R two ways: once using the three interior angles, and once using the exterior angle. Show both give the same measure.

→ w/ Triangle Sum Theorem: $50 + 60 + x = 180$
 $110 + x = 180$
 $x = 70$

15. An exterior angle of a triangle measures 130° , and one of its remote interior angles measures 70° . Find the other remote interior angle, and find the interior angle of the triangle next to the exterior angle.

$$\begin{array}{r} 130 = 70 + x \\ -70 \quad -70 \\ \hline 60 = x \end{array}$$

$$60^\circ = x$$

↑ other remote interior $\angle$

$$\begin{array}{r} 130 + y = 180 \text{ (linear pair)} \\ -130 \quad -130 \\ \hline y = 50^\circ \end{array}$$

$y = 50^\circ$ other interior angle

16. At vertex R of a triangle, the interior angle measures $(4x)^\circ$ and the exterior angle (along the extended side) measures $(5x)^\circ$. Write an equation using the fact that they form a **linear pair**, solve for x , and find the interior angle at R .

$$4x + 5x = 180 \text{ (linear pair)}$$

$$9x = 180$$

$$x = 20$$

$$\angle R = 4 \cdot 20 = \underline{80^\circ}$$

17. Which equation correctly represents the exterior angle theorem for a triangle whose exterior angle is e and whose two remote interior angles are a and b ?

→ (A) $e = a + b$ 2 remote interior $\angle$ s add to get the exterior $\angle$.

(B) $e = a - b$

(C) $e = 180 - a - b$

(D) $a + b + e = 180$

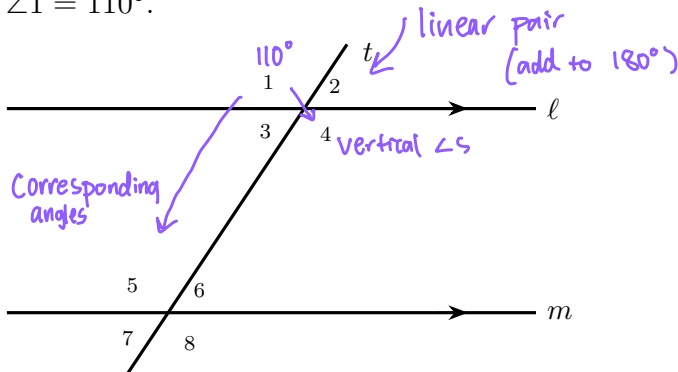
18. Explain why the exterior angle theorem must be true. Use the idea that an exterior angle and the interior angle next to it form a straight angle (180°), together with the triangle sum theorem.

If the third interior angle has to be a linear pair with the exterior $\angle$, they must add to 180° .

If that same interior $\angle$ is interior $\angle$ s w/ the other two remote interior $\angle$ s, they must all also add to 180° .

Questions 19-27 cover Artifact 10 (Parallel Lines and Transversals)

19. Lines ℓ and m are parallel and line t is a transversal. The eight angles are numbered. You are told that $\angle 1 = 110^\circ$.



Find $\angle 2$, $\angle 4$, and $\angle 5$. For each one, name its relationship to $\angle 1$.

$$110 + \angle 2 = 180 \quad \angle 2 = 70^\circ$$

$$\angle 4 = 110^\circ \text{ (vertical } \angle \text{s are equal)}$$

$$\angle 5 = 110^\circ \text{ (corresponding } \angle \text{s are equal)}$$

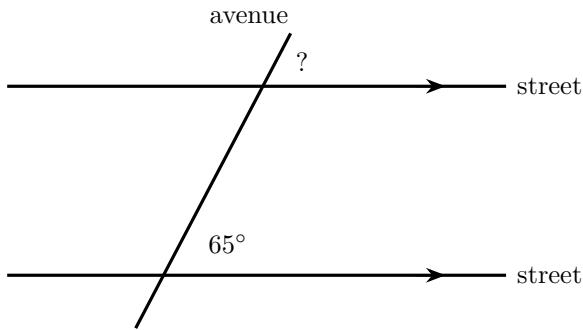
20. Lines ℓ and m are parallel and t is a transversal, with angles numbered as above. Complete the statement: $\angle 3$ and $\angle 5$ are which type of angle pair?

- (A) corresponding angles
 - (B) alternate interior angles
 - (C) same-side interior angles
 - (D) vertical angles
21. Two parallel lines are cut by a transversal. A pair of **corresponding** angles measure $(3x)^\circ$ and $(2x + 30)^\circ$. Write an equation, solve for x , and find the measure of the angle.

22. Two parallel lines are cut by a transversal. A pair of **same-side interior** angles measure $(2x)^\circ$ and $(3x + 20)^\circ$. Write an equation, solve for x , and find both angle measures.

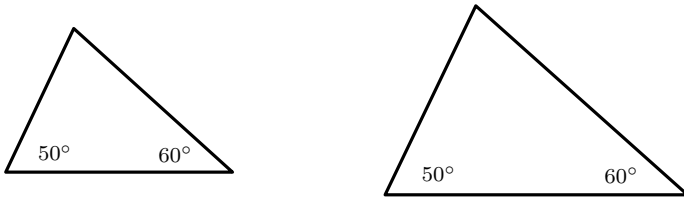
23. Two parallel lines are cut by a transversal. A pair of **alternate interior** angles measure $(4x - 10)^\circ$ and $(3x + 20)^\circ$. Write an equation, solve for x , and find the measure of the angle.

24. A straight avenue crosses two parallel streets, making a 65° angle at the lower street, as shown.



Find the angle marked ? at the upper street and name its relationship to the 65° angle.

25. Two triangles each have angles measuring 50° and 60° , as marked.



Find the third angle of each triangle. Then explain why the two triangles must be **similar**, naming the criterion you used.

26. Two triangles are similar by the angle-angle (AA) criterion. In the smaller triangle, the two shorter sides measure 3 and 4. In the larger triangle, the side corresponding to the side of length 3 measures 9. Write a proportion and solve for the length of the side corresponding to the side of length 4.

27. Explain why a straight transversal crosses every parallel line at the same angle. Use corresponding angles in your explanation.