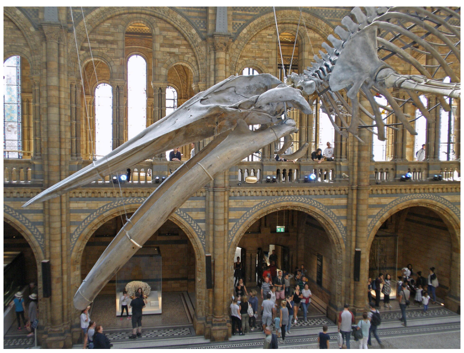
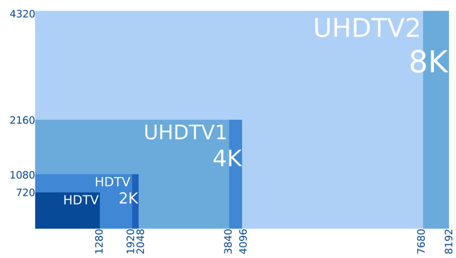
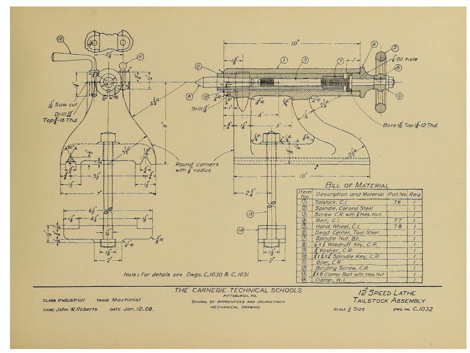
TOPIC OVERVIEW

Topic B — Dilations and Similarity

Scale Factor, Similar Figures, and Area & Perimeter



This overview guide contains...

The topic calendar — every objective and scheduled assessment across this part of the unit

Vocabulary — the terms you must build into your artifacts

Check Your Understanding problem sets — an extra tool to monitor your progress. Completing all questions is not required, but a guide kept up with, completed in full, and turned in at the end of the quarter raises your Standards Mastery Check score by **5%**. Answer keys for every question are posted on the class website.

Since this guide contains so much foundational information vital to success in this course, it is **essential** to keep up with it. It's THE **most important document** needed for success.

Topic Calendar

Monday, August 31st – Friday, September 11th

ARTIFACT 5 · Dilations and Similar Figures

WEEK 4 AUGUST 31–SEPTEMBER 4

BLOCK 1
MON–TUE
AUG 31–SEP 1

I can identify the vertices of an image transformed in a way that does not preserve congruence, when given the vertices or graph of the pre-image. **8.10B**

I can determine the proportional relationship of corresponding sides of similar figures, where the figures have a common side, even with a rotated or reflected image. **8.3A**

I can write a statement about the relationship of the side lengths of a dilated image given the graph of the pre-image. **8.3B**

ARTIFACT 6 · Algebraic Rules of Dilations

ARTIFACT 7 · Area and Perimeter of Dilations

BLOCK 2
WED–THU, SEP 2–3

I can determine the algebraic representation of a dilation when given coordinates and graphs of the pre-image and image, including only being given a graph of the pre-image and the ordered pair of a corresponding point on the graph of the image. **8.3C**

I can identify the ordered pair on the graph of a dilation when given the scale factor as a variable. **8.3C**

I can determine the ratios of area and perimeter of the pre-image and dilated image, including through expressions. **8.10D**

Feedback & Revision Studio · Performance Tasks 1.1 and 1.2
Friday, Sep 4 · also this day: Q1 Mathia Mid-Quarter Check

WEEK 5 SEPTEMBER 7–11

no school Monday

BLOCK 3
TUE–WED, SEP 8–9

Block 2, objectives 1 and 2, continued. **8.3C**

Performance Task 1.3 · Representing Scale Factor Algebraically
in class Tue–Wed, Sep 8–9 · revisions due Friday, Sep 18

BLOCK 4
THU–FRI, SEP 10–11

Block 2, objectives 1 and 2, continued. **8.3C**

Project Write-Up · Improve Dallas
due Friday, Sep 11

AFTER THE TOPIC

Unit 1 Mid-Unit Assessment · all TEKS to date (8.10A–D, 8.3A–C)
Friday, Sep 18 · revisions due Friday, Oct 2

Vocabulary

All of these terms must be eventually added to at least one artifact (Artifact 5, 6, and/or 7!)

pre-image	the original figure (e.g. $ABCD$)
image	the figure after the transformation, named with prime notation (e.g. $A'B'C'D'$)
corresponding sides/ angles/vertices	the matching parts of the pre-image and image
congruent/congruence	same size *and* shape; a dilation with a scale factor other than 1 is **not** congruent
rigid motion	a move that keeps the same size and shape (translation, reflection, rotation); a dilation is **not** a rigid motion
dilation	a transformation that resizes a figure by a scale factor from a center point; it keeps the same shape but changes the size
scale factor	the number every distance from the center is multiplied by; from the origin, $(x, y) \to (kx, ky)$ – e.g. $k = 2$ doubles, $k = \frac{1}{2}$ halves
center of dilation	the fixed point a figure is resized from (here, usually the origin)
origin	the center of the coordinate plane, point $(0, 0)$; the usual center of dilation
enlargement	a dilation with a scale factor greater than 1 (the image is larger)
reduction	a dilation with a scale factor between 0 and 1 (the image is smaller)
similar figures	figures with the same shape but not necessarily the same size: corresponding angles are equal and corresponding sides are proportional (symbol $\sim$)
similarity	the relationship between two similar figures
proportional	having a constant ratio; in a dilation every side of the image is the same number of times its matching side

ratio	a comparison of two quantities by division, e.g. $A'B' : AB = 2 : 1$
scale model	a real-world enlargement or reduction that keeps the same proportions as the original (e.g. a museum's blue-whale model)
perimeter	the distance around a figure; a dilation by scale factor k multiplies the perimeter by k
area	the amount of surface a figure covers; a dilation by scale factor k multiplies the area by k^2
algebraic representation (rule) of a dilation	the dilation written as mapping notation from the origin: $(x, y) \to (kx,\ ky)$
variable scale factor (k)	using a letter k in place of a number so one rule works for any size
expression	a math phrase that may use a variable (e.g. $4x$, k^2); area and perimeter ratios can be written as expressions
vertex/vertices	the corner point(s) of a figure
coordinate plane	a four-quadrant grid with two axes (x-values and y-values) on which transformations of figures are represented

Check Your Understanding

Topic B – Dilations and Similarity

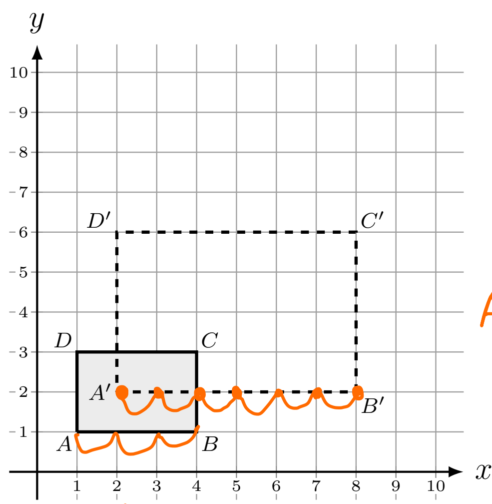
Scale Factor, Similar Figures, and Area & Perimeter

Questions 1-12 cover Artifact 5 (Dilations and Similar Figures)

1. In your own words, describe what a **dilation** does to a figure. Then explain why a dilation by a scale factor of 2 does *not* produce a congruent image.

A dilation enlarges or reduces a figure's size while maintaining the same shape. When a figure is dilated by a scale factor of 2, all side lengths are multiplied by 2, therefore no longer same size (not congruent).

2. Rectangle $ABCD$ is dilated from the origin to form rectangle $A'B'C'D'$, as shown.



A'B'=6

AB=3

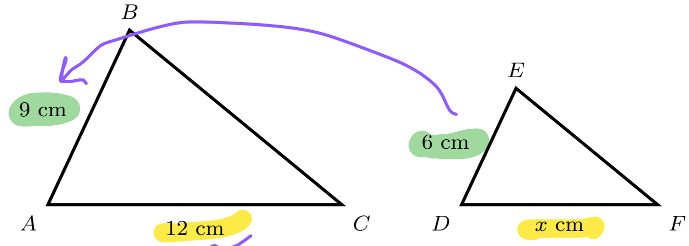
(a) Write the ratio of side $A'B'$ to side AB.

$\frac{A'B'}{AB} = \frac{6}{3} = 2$

(b) Is the image congruent to the pre-image? Explain how you know.

No, A'B' is twice as long as AB, so the image, with different side lengths, can no longer be congruent.

3. Triangle ABC and triangle DEF are similar. Side lengths are given in centimeters.



ABC DEF

Which proportion must be true?

~~(A)~~ $\frac{12}{x} = \frac{6}{9}$

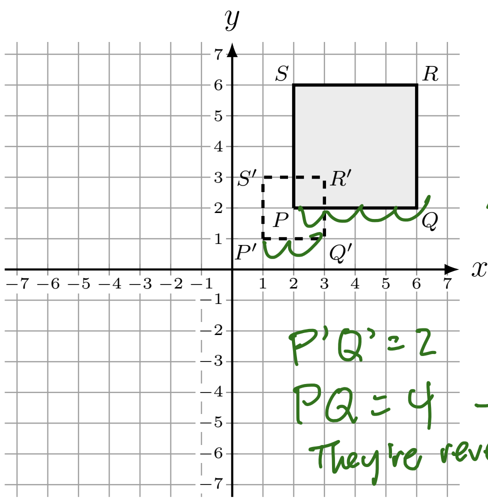
(B) $\frac{6}{9} = \frac{x}{12}$

~~(C)~~ $\frac{9}{6} = \frac{x}{12}$ different order

~~(D)~~ $\frac{6}{x} = \frac{12}{9}$ Corresponding sides don't match

*Corresponding sides must match in <u>same</u> <u>order</u>.

4. Quadrilateral $PQRS$ is dilated with the origin as the center of dilation to create quadrilateral $P'Q'R'S'$, as shown.



∠s stay the same

$P'Q' = 2$

$PQ = 4$

They're reversed

Which statement is true?

~~(A)~~ Quadrilateral $P'Q'R'S'$ is congruent to quadrilateral $PQRS$.

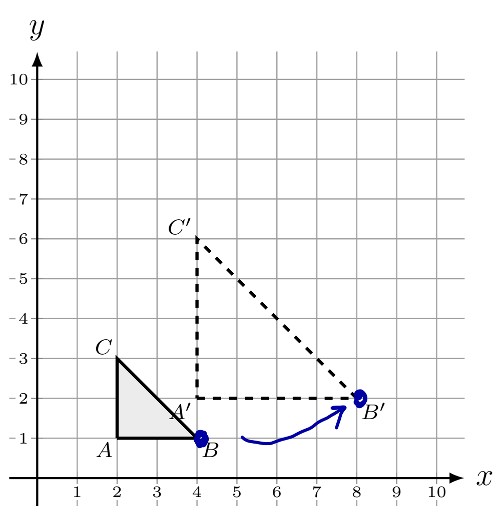
~~(B)~~ Each angle measure of quadrilateral $P'Q'R'S'$ is $\frac{1}{2}$ the corresponding angle measure of quadrilateral $PQRS$.

~~(C)~~ Each side length of quadrilateral $PQRS$ is $\frac{1}{2}$ the corresponding side length of quadrilateral $P'Q'R'S'$.

(D) Quadrilateral $P'Q'R'S'$ is <u>similar</u> to quadrilateral $PQRS$.

*Dilated figures are similar

5. Triangle ABC is dilated from the origin to form triangle $A'B'C'$, as shown.



(a) Which vertex of the image corresponds to vertex B, and what are its coordinates?

B' (8,2)

(b) Explain how you know this transformation does *not* preserve congruence.

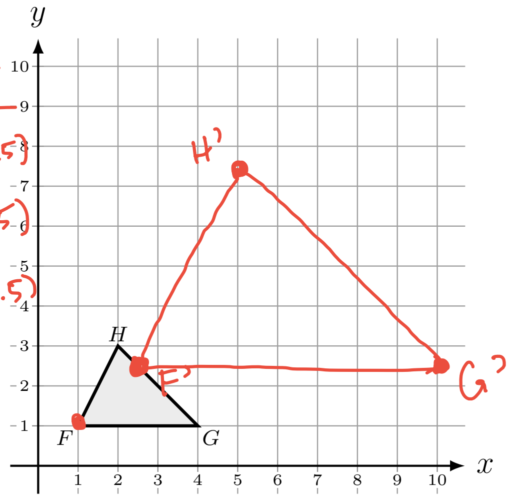
Side lengths change.

$AB = 2$

$A'B' = 4$

6. Triangle FGH is shown on the coordinate plane. Triangle FGH will be dilated with the origin as the center of dilation using the rule $(x, y) \to (2.5x,\ 2.5y)$ to create triangle $F'G'H'$.

Pre-image	Image
(1,1)	(2.5,2.5)
(2,3)	(5,7.5)
(4,1)	(10,2.5)



Which statement is true?

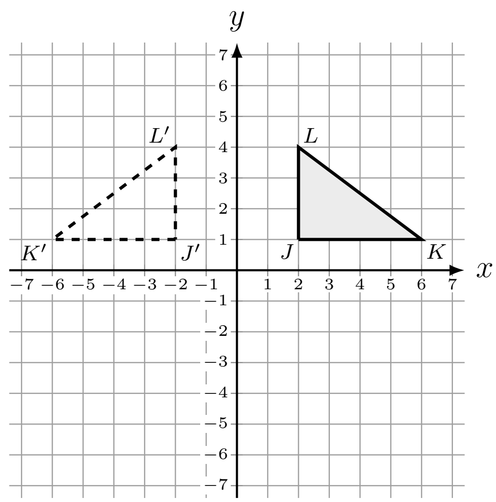
(A) Each angle measure of triangle $F'G'H'$ will be 2.5 times the corresponding angle measure of triangle FGH.

✓(B) Each side length of triangle $F'G'H'$ will be 2.5 times the corresponding side length of triangle FGH.

(C) Each angle measure of triangle $F'G'H'$ will be 2.5 more than the corresponding angle measure of triangle FGH.

(D) Each side length of triangle $F'G'H'$ will be 2.5 more than the corresponding side length of triangle FGH.

7. Triangle JKL and its image $J'K'L'$ are shown on the coordinate plane.



*Reflection across the y-axis → (x,y) → (-x,y)

Which rule describes the transformation?

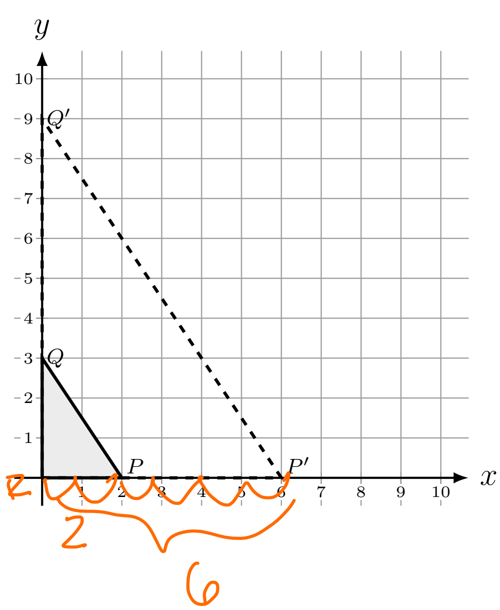
(A) $(x, y) \to (x - 8,\ y)$

(B) $(x, y) \to (x,\ -y)$

→ (C) $(x, y) \to (-x,\ y)$

(D) $(x, y) \to (-x,\ -y)$

8. Two similar triangles share the same vertex at the origin, as shown. The smaller triangle is the pre-image and the larger triangle is its dilation.



(a) Write the ratio of a side of the larger triangle to its corresponding side of the smaller triangle.

$\frac{P'R'}{PR} = \frac{6}{2} = 3$

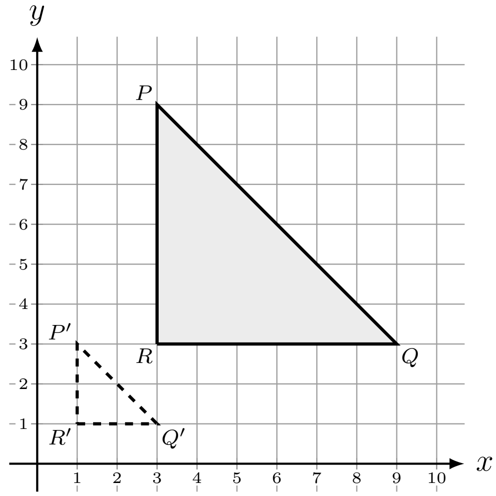
(b) What is the scale factor of the dilation?

$K = 3$

9. Triangle PQR is dilated with the center of dilation at the origin to form triangle $P'Q'R'$, as shown.

$R(3,3)$

$R'(1,1)$



goes from bigger to smaller, not smaller to bigger

Which statement must be true?

~~(A)~~ Triangle PQR was dilated by a scale factor of 3 to create triangle $P'Q'R'$.

(B) $P'Q' = \frac{1}{3}(PQ)$

~~(C)~~ $\dfrac{PQ}{P'Q'} = \dfrac{Q'R'}{QR}$ ← mismatched

~~(D)~~ Triangle PQR is congruent to triangle $P'Q'R'$.

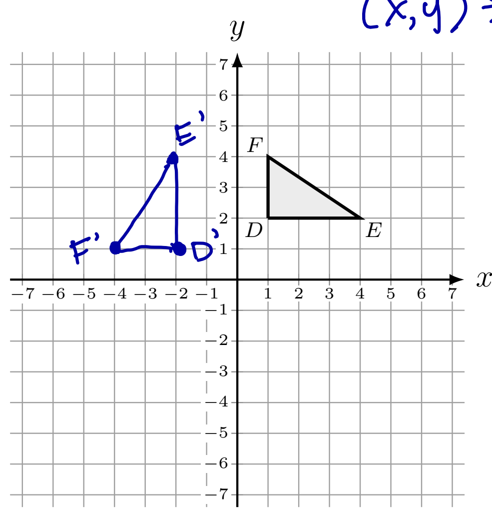
10. Determine whether each transformation produces a **congruent** image or a **similar but not congruent** image. Place a check in the correct column for each row.

	Congruent image	**Similar but not congruent**
A dilation by a scale factor of 2	☐	☑
A reflection across the x-axis	☑	☐
A dilation by a scale factor of $\frac{1}{2}$	☐	☑
A rotation of 90° about the origin	☑	☐

11. Triangle DEF has vertices $D(1,2)$, $E(4,2)$, and $F(1,4)$. Rotate triangle DEF 90° counterclockwise about the origin. Plot the image $D'E'F'$ on the grid, label each vertex with its coordinates, and write the algebraic rule for the rotation.

$(x,y) \to (-y,x)$

Pre-image	Image
D (1,2)	D' (-2,1)
E (4,2)	E' (-2,4)
F (1,4)	F' (-4,1)

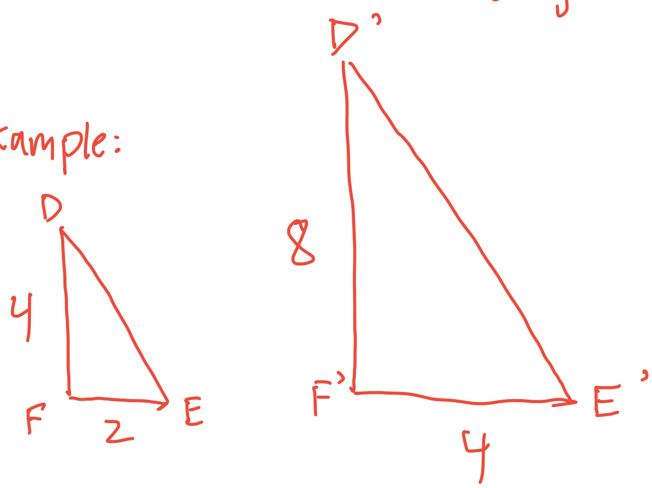


12. A student claims that two similar figures must be congruent. Decide whether this is correct, and justify with a labeled dilation example that refers to corresponding angles, corresponding sides, and the scale factor.

According to the definition, similar figures must have the same shape, where corresponding sides are proportional & corresponding angles equal.

This means that they can be congruent, but they don't have to be. If the scale factor of a dilated (similar) figure is anything other than 1, then it is still similar but not congruent.

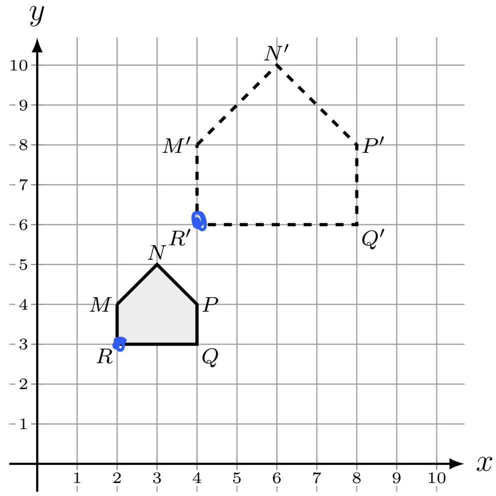
Example:



$$\frac{D'F'}{DF} = \frac{4}{2} = 2 \checkmark \qquad \frac{E'F'}{EF} = \frac{4}{2} = 2 \checkmark$$

$K = 2$, but $\triangle DEF$ is not congruent to $\triangle D'E'F'$ as $\triangle D'E'F'$'s side lengths are double.

13. Pentagon $MNPQR$ is dilated with the origin as the center of dilation to create pentagon $M'N'P'Q'R'$, as shown.



	Pre-image	Image		
R	(2,3)	(4,6)	$\frac{6}{3}=2$	$k=2$

Which rule best represents the dilation that was applied to pentagon $MNPQR$ to create pentagon $M'N'P'Q'R'$?

(A) $(x, y) \to (2x,\ 2y)$

~~(B)~~ $(x, y) \to (x,\ y+2)$

~~(C)~~ $(x, y) \to (x+2,\ y+2)$

~~(D)~~ $(x, y) \to \left(\frac{1}{2}x,\ \frac{1}{2}y\right)$

14. Write the algebraic rule for each dilation from the origin. Write your rule in the parentheses.

(a) scale factor of 2

$(x, y) \to (\ 2x\ ,\ 2y\ )$

(b) scale factor of 3

$(x, y) \to (\ 3x\ ,\ 3y\ )$

(c) scale factor of $\frac{1}{2}$

$(x, y) \to (\ \frac{1}{2}x\ ,\ \frac{1}{2}y\ )$

(d) scale factor of 5

$(x, y) \to (\ 5x\ ,\ 5y\ )$

15. Triangle MNP is graphed on a coordinate plane. The triangle is dilated with the origin as the center of dilation to create triangle $M'N'P'$. The table shows the coordinates of the original vertices and the corresponding coordinates of the new vertices after the dilation is applied.

*Pick 2 corresponding points

$\frac{\text{image}}{\text{pre-image}}$

Vertices of $\triangle MNP$	Vertices of $\triangle M'N'P'$
$(-4, 6)$	$(-10, 15)$
$(2, 6)$	$(5, 15)$
$(-4, -2)$	$(-10, -5)$

$k = \frac{15}{6} = \frac{5}{2}$

$5 \div 2 = 2.5$

Which rule represents the dilation that was applied to triangle MNP to create triangle $M'N'P'$?

~~(A)~~ $(x, y) \to \left(\frac{2}{5}x,\ \frac{2}{5}y\right)$

(B) $(x, y) \to (2.5x,\ 2.5y)$

~~(C)~~ $(x, y) \to (x-6,\ y+9)$

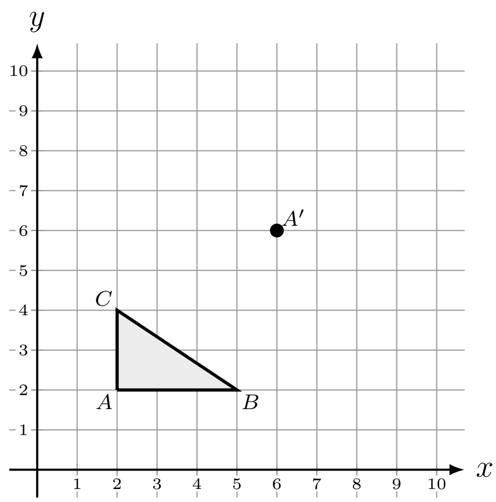
~~(D)~~ $(x, y) \to (6x,\ 9y)$

$k = 2.5$

16. Triangle ABC has vertices $A(2,2)$, $B(5,2)$, and $C(2,4)$. After a dilation from the origin, only one image point was recorded: $A'(6,6)$, shown as the dot.

Pre-image	Image
(2, 2)	(6, 6)

$k = \frac{6}{2} = 3$



(a) Use A and A' to find the scale factor, then write the dilation rule.

$(x, y) \to ($ $3x$, $3y$ $)$

(b) Use your rule to find the coordinates of B' and C'.

$\times 3$ $\times 3$

$C(2,4) \to C'(6, 12)$

$B(5,2) \to B'(15, 6)$

17. The table shows the coordinates of the vertices of pentagon $ABCDE$.

x	y
-1	1
1	6
4	1
2	-5
-6	-2

Pentagon $ABCDE$ is dilated by a scale factor of $\frac{7}{3}$ with the origin as the center of dilation to create pentagon $A'B'C'D'E'$. If (x, y) represents the location of any point on pentagon $ABCDE$, which ordered pair represents the location of the corresponding point on pentagon $A'B'C'D'E'$?

~~(A)~~ $\left(\frac{3}{7}x, \frac{3}{7}y\right)$

~~(B)~~ $\left(x + \frac{7}{3}, y + \frac{7}{3}\right)$

~~(C)~~ $\left(x + \frac{3}{7}, y + \frac{3}{7}\right)$

→ (D) $\left(\frac{7}{3}x, \frac{7}{3}y\right)$ Scale factor means always multiply.

18. A figure is dilated from the origin by a scale factor of k.

(a) Write the coordinates of the image of point $P(5,2)$ in terms of k.

$P'(5 \cdot k, 2 \cdot k)$ $(5k, 2k)$

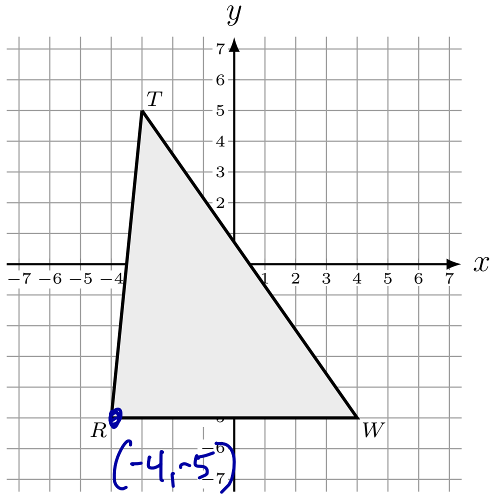
(b) A design requires the image of point $B(5,2)$ to land at $(15,6)$. What value of k produces this, and how do you know?

Pre-image	Image
$B(5,2)$	$B'(15,6)$

$k = \frac{6}{2} = 3$

*go backwards to find k → divide

19. Triangle TRW is shown on the coordinate grid.



Triangle TRW will be dilated by a scale factor of n with the origin as the center of dilation to create triangle $T'R'W'$. Which ordered pair best represents the location of point R'?

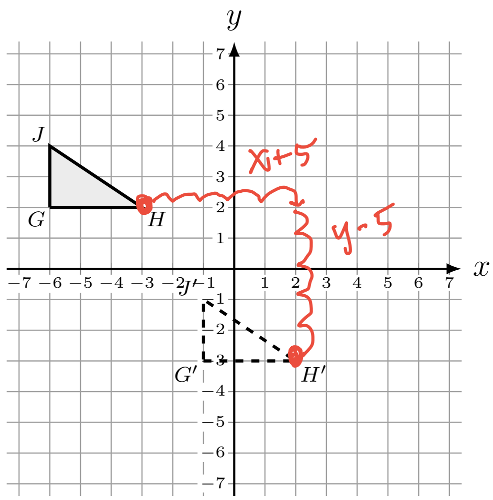
~~(A)~~ $\left(\frac{-4}{n}, \frac{-5}{n}\right)$

→ (B) $(-4n, -5n)$ ← multiply!

~~(C)~~ $(-4+n, -5+n)$

~~(D)~~ $(-4-n, -5-n)$

20. Triangle GHJ and its image $G'H'J'$ are shown on the coordinate plane.



*not a dilation!!

→ Translation

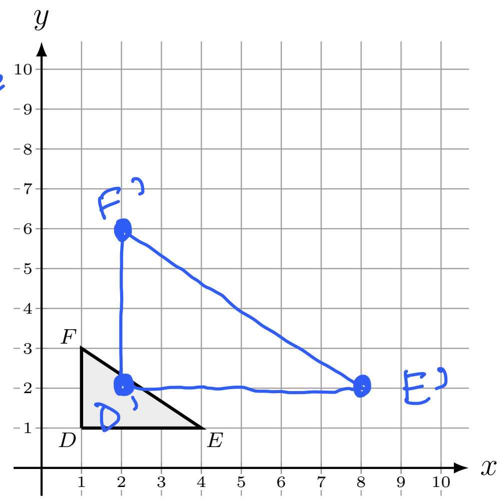
Write the algebraic rule for the transformation, and name the transformation your rule represents.

$(x, y) \rightarrow ($ x+5 , y-5 $)$

21. Triangle DEF with vertices $D(1,1)$, $E(4,1)$, and $F(1,3)$ is dilated from the origin by a scale factor of 2. Plot the image $D'E'F'$ on the grid below, and label the coordinates of each vertex.

x2 for every coordinate

Pre-image	Image
D (1,1)	D' (2,2)
E (4,1)	E' (8,2)
F (1,3)	F' (2,6)



22. The table describes two dilations with the center of dilation at the origin. For each row, mark the rule that describes the dilation. Each rule may be used once or not at all.

	$(2x,\ 2y)$	$\left(\frac{1}{2}x,\ \frac{1}{2}y\right)$	$\cancel{(x+2,\ y+2)}$	$\cancel{\left(x-\frac{1}{2},\ y-\frac{1}{2}\right)}$
A triangle is dilated by a scale factor of $\frac{1}{2}$.	☐	☐ ✓	☐	☐
A square is dilated by a scale factor of 2.	☐ ✓	☐	☐	☐

23. A figure on the coordinate plane is transformed by the rule $(x, y) \to (-y,\ x)$. Which transformation does the rule represent?

Rotation
270° clockwise -or-
90° Counterclockwise

→ (A) A rotation of 90° counterclockwise about the origin

~~(B) A rotation of 180° about the origin~~ $(x,y) \to (-x,-y)$

~~(C) A reflection across the y-axis~~

~~(D) A translation left and up~~

24. One point of a figure is at $(2, 2)$, and its image after a transformation is at $(4, 4)$. One student writes the rule $(x, y) \to (x+2,\ y+2)$. Another student writes the rule $(x, y) \to (2x,\ 2y)$. Explain why both rules fit this one point, and describe what additional information you would need to decide which rule is correct.

Because $2 \cdot 2 = 4$ & $2+2=4$, so both rules work. To be sure of which one, we'd need to know if it's talking about a translation or dilation.

25. A rectangle is 3 units wide and 2 units tall. It is dilated from the origin by a scale factor of 2.

    (a) Find the perimeter of the original rectangle and of the dilated image, then write the ratio (image : original).

    P = 2·3 + 2·2
    6 + 4 = 10

    3
    2 $\boxed{2\cdot 3 = 6}$ $\xrightarrow{\times 2}$
    6
    4 $\boxed{6\cdot 4 = 24}$

    Original perimeter: 10 units
    Dilated: 20
    P = 2·6 + 2·4 = 20
    20 : 10 $\frac{20}{10} = 2$

    (b) Find the area of each rectangle and write that ratio (image : original).

    image : original
    24 : 6 $\frac{24}{6} = 4$

26. A figure is dilated from the origin by a scale factor of 3. The area of the image is how many times the area of the original figure?

    ~~(A)~~ 3 times

    ~~(B)~~ 6 times

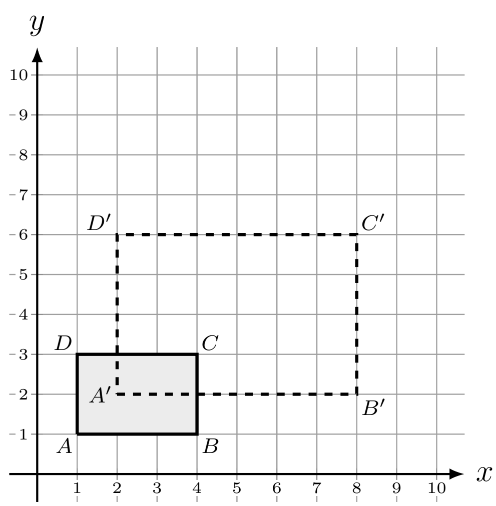
    → (C) 9 times

    ~~(D)~~ 12 times

    It is always a ratio of $k^2 : 1$, so in this case, 3^2 or 9 times the area of the original.

27. Rectangle $ABCD$ is dilated from the origin by a scale factor of 2 to form $A'B'C'D'$, as shown.

    k = 2



Find the perimeter ratio and the area ratio ($A'B'C'D'$: $ABCD$). Show the measures you used.

Perimeter ratio = k = 2
Area ratio = k^2 = 4

28. Complete the table for a dilation by scale factor k. Write each ratio as image : original. The bottom row uses the variable k.

Scale factor	Perimeter ratio	Area ratio
$k = 2$	2	$2^2 = 4$
$k = 3$	3	$3^2 = 9$
$k = 5$	5	$5^2 = 25$
k (any)	k	k^2

29. A figure is dilated with the origin as the center of dilation by a scale factor of 5. The perimeter of the image is how many times the perimeter of the original figure?

(A) 5 times

~~(B)~~ 10 times

~~(C)~~ 20 times

~~(D)~~ 25 times

30. A figure is dilated from the origin by a scale factor of $\frac{1}{2}$. What happens to its area? Write the area ratio (image : original) and explain your reasoning.

Area is $\left(\frac{1}{2}\right)^2$ times the amount:

$$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

31. The area of a dilated image is 16 times the area of the original figure. What scale factor was used?

~~(A)~~ 2

(B) 4

~~(C)~~ 8

~~(D)~~ 16

$$k^2 = 16$$

$$\sqrt{16} = 4, \quad k = 4$$

32. A square has a side length of s. It is dilated from the origin by a scale factor of k. Write an expression for the **perimeter** of the image and an expression for the **area** of the image, each in terms of s and k.

Perimeter $= s + s + s + s = 4s$

Area $= k^2 \cdot 4s$

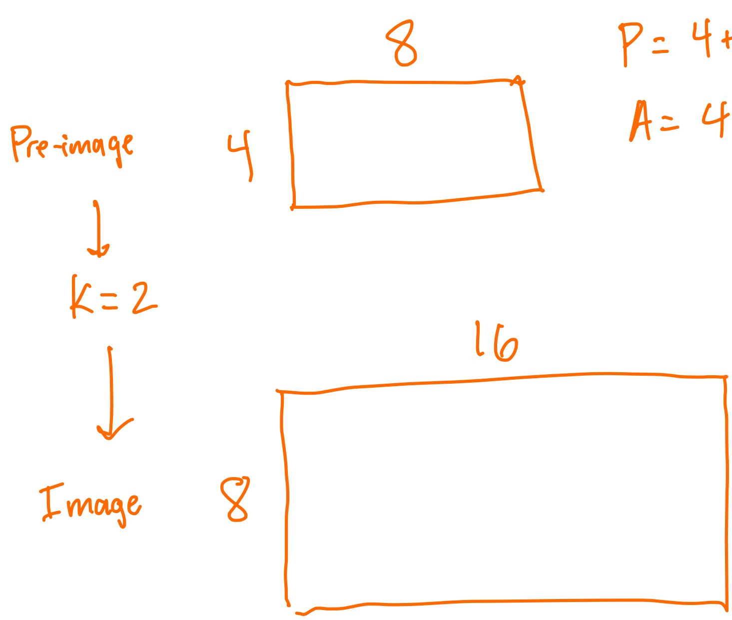
33. For a dilation by scale factor k, determine whether each quantity is multiplied by k or by k^2. Place a check in the correct column for each row.

	Multiplied by k	**Multiplied by k^2**
The perimeter of the figure	☑	☐
The area of the figure	☐	☑
The length of one side	☑	☐
The distance from the origin to a vertex	☑	☐

34. A triangle is reflected across the x-axis. A classmate says the image must have a smaller area because the transformation moved it below the axis. Decide whether the classmate is correct. In your answer, state what happens to the perimeter and to the area of a figure under a reflection, and contrast that with what happens under a dilation.

The Student is incorrect. An image after a reflection is still congruent to the pre-image, even when orientation changes. Under a dilation, with a scale factor less than 1, area would decrease.

35. Explain why dilating a figure by a scale factor of k multiplies its **perimeter** by k but its **area** by k^2. Support your answer with a worked numerical example.



$P = 4+8+4+8 = 24$

$A = 4 \cdot 8 = 32$

$P = 8+16+8+16 = 48 \ (2x)$

$A = 16 \cdot 8 = 128 \ (4x)$

x2 x2

When you multiply by a scale factor, and then multiply the side lengths again for area, you are essentially multiplying by the scale factor twice, hence the ratio k^2.