

# Topic Overview

## Topic A — Rigid Motions

Translations, Reflections, and Rotations



### This overview guide contains...

**The topic calendar** — every objective and scheduled assessment across this part of the unit

**Vocabulary** — the terms you must build into your artifacts

**Check Your Understanding problem sets** — an extra tool to monitor your progress. Completing all questions is not required, but a guide kept up with, completed in full, and turned in at the end of the quarter raises your Standards Mastery Check score by **5%**. Answer keys for every question are posted on the class website.

Since this guide contains so much foundational information vital to success in this course, it is **essential** to keep up with it. It's THE **most important document** needed for success.

## Topic Calendar

Tuesday, August 11<sup>th</sup> – Friday, August 28<sup>th</sup>

### ARTIFACT 1 · Orientation and Congruence

#### WEEK 1 AUGUST 10–14

no school Monday

##### BLOCK 1

TUE–WED, AUG 11–12

I can describe a transformation (rotation, reflection, translation) and its effect in terms of side lengths, angle measures, or figure orientation when given a graph of the pre-image. **8.10A**

##### BLOCK 2

THU–FRI, AUG 13–14

I can identify a transformation when given the effects on a figure. **8.10A**

I can describe a transformation that does or does not preserve congruence. **8.10B**

### ARTIFACT 2 · Translations

#### WEEK 2 AUGUST 17–21

##### BLOCK 3

MON–TUE, AUG 17–18

I can determine the algebraic representation of a translation when given the graph of the image and pre-image. **8.10C**

### ARTIFACT 3 · Reflections

##### BLOCK 4

WED–THU, AUG 19–20

I can identify whether an algebraic representation of a transformation preserves congruence. **8.10B**

I can determine the algebraic representation of a reflection when given the graph of the image and pre-image. **8.10C**

Block 3 objective, continued. **8.10C**

### Performance Task 1.1 · Describing Orientation and Congruence

in class Friday, Aug 21 · Feedback & Revision Studio Friday, Sep 4

### ARTIFACT 4 · Rotations

#### WEEK 3 AUGUST 24–28

##### BLOCK 5

MON–TUE, AUG 24–25

I can identify a transformation as a translation or reflection when given the algebraic representation. **8.10C**

I can determine the algebraic representation of a clockwise and counterclockwise rotation about the origin when given a verbal description of the rotation. **8.10C**

Block 4, objective 2, continued. **8.10C**

**Site Map Selection · Improve Dallas**

introduced Mon–Tue, Aug 24–25 · due Friday, Aug 28

**BLOCK 6**

WED–THU, AUG 26–27

Block 5, objective 2, continued. **8.10C**

I can determine the algebraic representation of a rotation about the origin when given the graph of the image and pre-image. **8.10C**

**Performance Task 1.2 · Algebraic Representations of Transformations**

Friday, Aug 28 · Feedback &amp; Revision Studio Friday, Sep 4

**Q1 Participation · Weeks 1–3** Friday, Aug 28

## Vocabulary

*All of these terms must be eventually added to at least one artifact (Artifact 1, 2, 3, and/or 4!)*

|                                            |                                                                                                                  |
|--------------------------------------------|------------------------------------------------------------------------------------------------------------------|
| <b>transformation</b>                      | a rule that moves or resizes a figure on the coordinate plane                                                    |
| <b>pre-image</b>                           | the original figure (e.g. $ABCD$ )                                                                               |
| <b>image</b>                               | the figure after the transformation, named with prime notation (e.g. $A'B'C'D'$ )                                |
| <b>prime notation</b>                      | the notation used with apostrophes to notate the image (new figure) (e.g. $A'B'C'D'$ or $P'Q'R'S'$ )             |
| <b>rigid motion</b>                        | a move that keeps the same size and shape (translation, reflection, or rotation)                                 |
| <b>congruent/congruence</b>                | same size and shape; equal side lengths and equal angle measures                                                 |
| <b>corresponding sides/angles/vertices</b> | the matching parts of the pre-image and image                                                                    |
| <b>orientation</b>                         | the order of the vertices (clockwise vs. counterclockwise); a reflection reverses orientation                    |
| <b>preserve</b>                            | to keep unchanged (rigid motions preserve congruence)                                                            |
| <b>translation</b>                         | a slide across the coordinate plane; every point moves the same distance in the same direction                   |
| <b>reflection</b>                          | a flip across a line – produces mirror image and reverses the order of vertices (clockwise vs. counterclockwise) |
| <b>line of reflection</b>                  | the mirror line (here, usually the x-axis or y-axis)                                                             |
| <b>rotation</b>                            | a turn about a fixed point                                                                                       |
| <b>center of rotation</b>                  | the fixed point of a turn (here, usually the origin)                                                             |
| <b>origin</b>                              | the center of the four-quadrant coordinate plane with coordinate point $(0,0)$                                   |

|                                        |                                                                                                                                                                           |
|----------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>clockwise</b>                       | a turn to the right on the coordinate plane                                                                                                                               |
| <b>counterclockwise</b>                | a turn to the left on the coordinate plane                                                                                                                                |
| <b>algebraic representation (rule)</b> | the move of a transformation written as mapping notation                                                                                                                  |
| <b>mapping notation</b>                | the way algebraic representations are written:<br>$(x, y) \rightarrow (x - 2, y + 1)$                                                                                     |
| <b>coordinate plane</b>                | a four-quadrant grid with two axes (number sets – x-values and y-values), creating a 2-D representation for which transformations of geometric figures can be represented |
| <b>x-axis</b>                          | the boundary line in the coordinate plane that separates positive y-values (top two quadrants) from negative ones (bottom two quadrants)                                  |
| <b>y-axis</b>                          | the boundary line in the coordinate plane that separates positive x-values (right two quadrants) from negative ones (left two quadrants)                                  |
| <b>quadrant</b>                        | a region representing one-fourth of the coordinate grid                                                                                                                   |
| <b>dilation</b>                        | a resize by a scale factor (not a rigid motion, does not preserve congruence)                                                                                             |

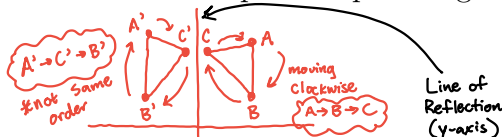
# Check Your Understanding

## Topic A – Rigid Motions

### Translations, Reflections, and Rotations

Questions 1-11 cover Artifact 1 (Orientation and Congruence)

1. Draw an example of a pre-image and image that are **congruent** but have different **orientations**.



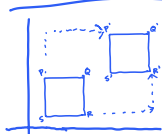
$\triangle ABC$  is congruent to  $\triangle A'B'C'$  as they have the same size & shape. However, the order of the vertices (in clockwise order, shown to the left) is different between the pre-image & image.

2. How can an image be congruent and still have a different orientation? Use the words corresponding sides and corresponding angles in your response.

As shown above, despite  $\triangle A'B'C'$ 's change in the order of the vertices, corresponding sides, such as  $\overline{AC}$  &  $\overline{A'C'}$ , still have the exact same length. Corresponding angles, like  $\angle B$  &  $\angle B'$ , are in equal measure as well, and the figures overall are congruent (same size, same shape).

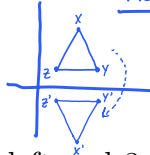
3. List the transformations that preserve congruence and draw an example of each.

#### Translations



\*Still Congruent

#### Reflections

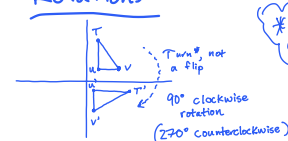


Reflection across the x-axis

\*Orientation changes

\*Still Congruent

#### Rotations



Turn, not a flip  
90° clockwise rotation  
(270° counterclockwise)

\*Still Congruent

4. Triangle  $RST$  is translated 5 units left and 3 units down to form triangle  $R'S'T'$ . Which statement is true?

\*translation → preserves congruence

- (A) The side lengths of triangle  $R'S'T'$  are greater than the corresponding side lengths of triangle  $RST$ .  
No, congruent side lengths after translation

- (B) Triangle  $R'S'T'$  is not congruent to triangle  $RST$ .

- (C) The angle measures of triangle  $R'S'T'$  are equal to the corresponding angle measures of triangle  $RST$ .  
Yes, congruent  $\angle$  measures

- (D) The orientation of triangle  $R'S'T'$  is the reverse of the orientation of triangle  $RST$ .

No, only reflections reverse orientation.

Same size, same shape

5. Pentagon  $JKLMN$  is reflected across the  $y$ -axis to form pentagon  $J'K'L'M'N'$ . Which statement is true?

\* Reflection → preserves congruence

- (A) The area of pentagon  $J'K'L'M'N'$  is not equal to the area of pentagon  $JKLMN$ .  
 (B) Pentagon  $J'K'L'M'N'$  is not congruent to pentagon  $JKLMN$ .  
 (C) The perimeter of pentagon  $J'K'L'M'N'$  is greater than the perimeter of pentagon  $JKLMN$ .  
 (D) The angle measures of pentagon  $J'K'L'M'N'$  are congruent to the corresponding angle measures of pentagon  $JKLMN$ .

Same size = same area

Same size = same side lengths = same perimeter

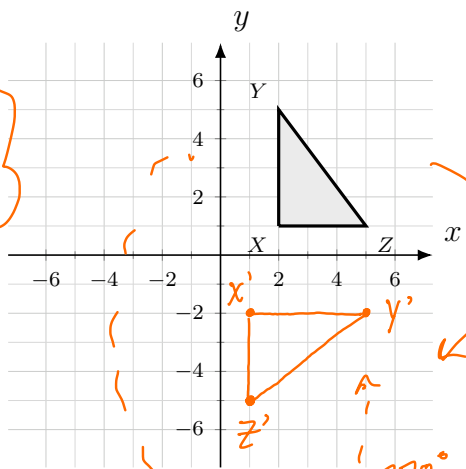
Yes ✓

6. Triangle  $XYZ$  is rotated  $90^\circ$  clockwise about the origin to form triangle  $X'Y'Z'$ .

Which statement is true?

- (A) The sum of the angle measures of triangle  $XYZ$  is  $90^\circ$  less than the sum of the angle measures of triangle  $X'Y'Z'$ .  
 (B) The angle measures of triangle  $XYZ$  are less than the corresponding angle measures of triangle  $X'Y'Z'$ .  
 (C) Triangle  $XYZ$  is not congruent to triangle  $X'Y'Z'$ .

- (D) The area of triangle  $XYZ$  is equal to the area of triangle  $X'Y'Z'$ .



\* Rotation: Preserves congruence

90° clockwise

270° counterclockwise

∠s are congruent

congruent → equal areas

7. A figure on the coordinate plane is transformed to create a new figure. Determine whether each transformation preserves or does not preserve congruence. Place a check in the correct column for each row.

| * See #3 → only reflections, translations, & rotations apply!               | Preserves Congruence                | Does Not Preserve Congruence        |
|-----------------------------------------------------------------------------|-------------------------------------|-------------------------------------|
| A reflection across the $x$ -axis                                           | <input checked="" type="checkbox"/> | <input type="checkbox"/>            |
| A rotation of $270^\circ$ about the origin                                  | <input checked="" type="checkbox"/> | <input type="checkbox"/>            |
| A dilation by a scale factor of 2 with the origin as the center of dilation | <input type="checkbox"/>            | <input checked="" type="checkbox"/> |
| A translation 6 units right and 4 units up                                  | <input checked="" type="checkbox"/> | <input type="checkbox"/>            |

\* Changes size!

8. A figure is reflected across a line. Three of the statements below about the figure and its image are true. Which statement does **not** belong?

- (A) The corresponding side lengths are equal.  
 (B) The corresponding angle measures are equal.  
 (C) The figure and its image are congruent.  
 (D) The orientation of the vertices stays the same.

Reflections preserve congruence

Reflections reverse orientation.

Rotations → preserve congruence

9. Quadrilateral  $WXYZ$  is rotated  $180^\circ$  about the origin to form quadrilateral  $W'X'Y'Z'$ . Which statement best describes how the two figures compare?

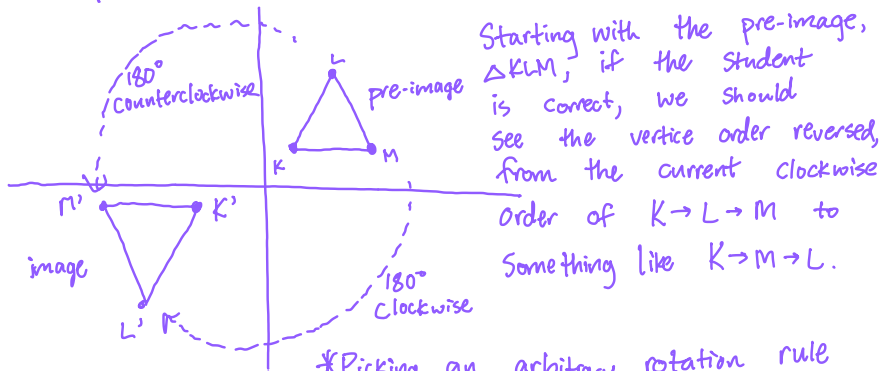
- (A) The figures are congruent, and the corresponding angle measures are equal. ✓
- (B) The figures are congruent, but the side lengths of the image are doubled. If side lengths change, no longer can be congruent
- (C) The figures are not congruent because the orientation changed.
- (D) The figures are not congruent because the area changed. (Orientation doesn't change congruence)

10. A transformation maps a figure to an image with all corresponding side lengths equal and all corresponding angle measures equal, but the order of the vertices is reversed. Name every transformation this could be, and explain how you know.

This is a reflection. When a 2-D figure flips, it creates a mirror image, which reverses the orientation (order of the vertices). No other transformation preserves congruence while also changing the vertices order.

11. A student claims that whenever a figure and its image are congruent but the orientation is reversed, the transformation must be a rotation. Decide whether the student is correct. Use a labeled example to justify your reasoning, and in your explanation name the transformation and describe its effect on side lengths, angle measures, and orientation.

\*Example of rotation



\*Picking an arbitrary rotation rule ( $180^\circ$ ) we can compare the order in which the vertices (image) are arranged

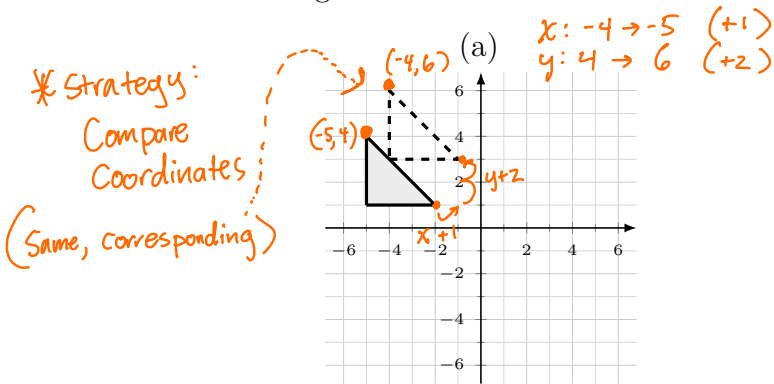
Image vertices order:  $K' \rightarrow L' \rightarrow M'$   
(clockwise)

Verdict: no change in vertices clockwise order  
↓  
no change in orientation



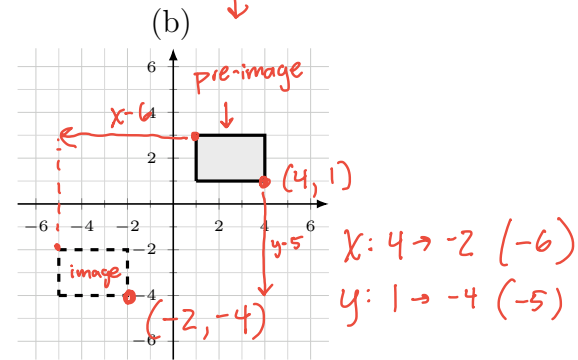
Questions 12-22 cover Artifact 2 (Translations)

12. For each translation shown, the shaded figure is the pre-image and the dashed figure is its image. Write the algebraic rule on the line under each graph.



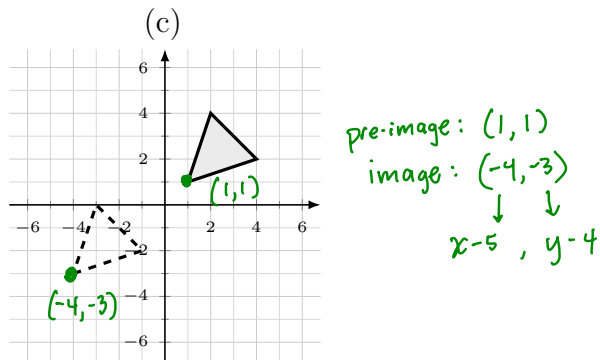
Algebraic Rule:

$$(x, y) \rightarrow (x + 1, y + 2)$$



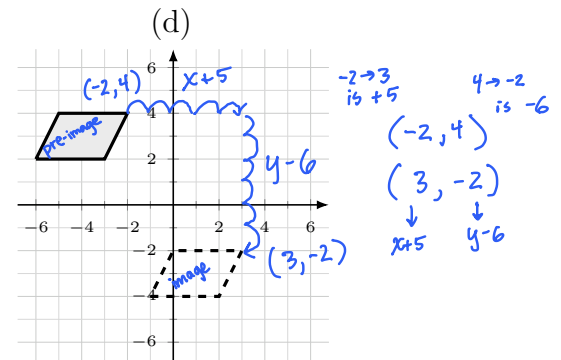
Algebraic Rule:

$$(x, y) \rightarrow (x - 6, y - 5)$$



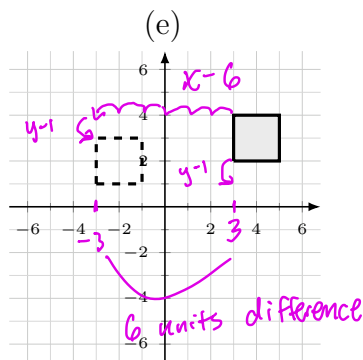
Algebraic Rule:

$$(x, y) \rightarrow (x - 5, y - 4)$$



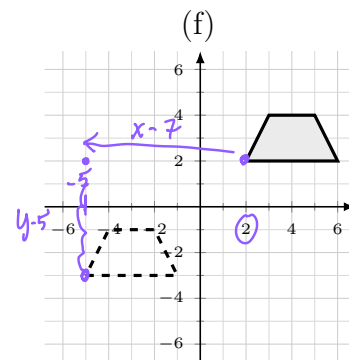
Algebraic Rule:

$$(x, y) \rightarrow (x + 5, y - 6)$$



Algebraic Rule:

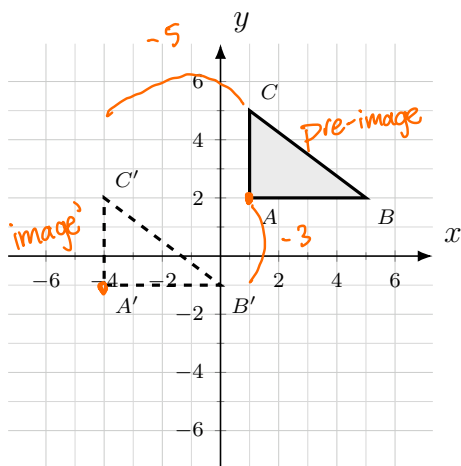
$$(x, y) \rightarrow (x - 6, y - 1)$$



Algebraic Rule:

$$(x, y) \rightarrow (x - 7, y - 5)$$

13. Triangle  $ABC$  is translated to form triangle  $A'B'C'$ , as shown.



$A \rightarrow A' \quad (1, 2) \rightarrow (-4, -1)$

- (a) Write the algebraic rule for the translation.

$(x, y) \rightarrow (x - 5, y - 3)$

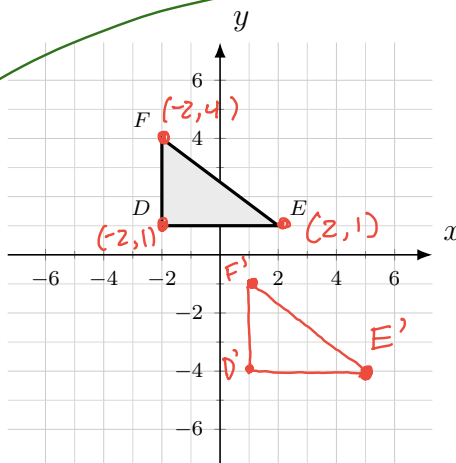
- (b) Describe the translation in words (how many units, and in which directions).

$\triangle ABC$  (pre-image) is translated 5 units left & 3 units down to create image  $\triangle A'B'C'$ .

14. Triangle  $DEF$  with vertices  $D(-2, 1)$ ,  $E(2, 1)$ , and  $F(-2, 4)$  is translated by the rule  $(x, y) \rightarrow (x + 3, y - 5)$ . Plot the image  $D'E'F'$  on the grid below, and label the coordinates of each vertex.

Add 3 to every X-value

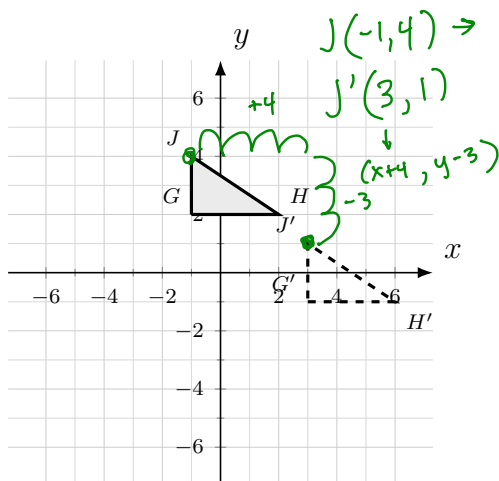
Subtract 5 from every y-value



New Coordinates

$D'(1, -4)$   
 $F'(1, -1)$   
 $E'(5, -4)$

15. Triangle  $GHJ$  is translated to form triangle  $G'H'J'$ , as shown.



Which rule represents the translation that maps triangle  $GHJ$  to triangle  $G'H'J'$ ?

- (A)  $(x, y) \rightarrow (x + 4, y - 3)$   
~~(B)  $(x, y) \rightarrow (x - 4, y + 3)$~~   
~~(C)  $(x, y) \rightarrow (x + 3, y - 4)$~~   
~~(D)  $(x, y) \rightarrow (x + 4, y + 3)$~~

16. Write the algebraic rule for each translation described. Write your rule in the parentheses.

(a) 3 units  $\overset{x+}{\text{right}}$  and 2 units  $\overset{y+}{\text{up}}$   
 $(x, y) \rightarrow (x+3, y+2)$

(b) 5 units  $\overset{x-}{\text{left}}$  (no  $y$ -change)  
 $(x, y) \rightarrow (x-5, y)$

(c) 4 units down (no  $x$ -change)  
 $(x, y) \rightarrow (x, y-4)$

(d) 6 units  $\overset{x-}{\text{left}}$  and 1 unit down  
 $(x, y) \rightarrow (x-6, y-1)$

17. Parallelogram  $KLMN$  has vertices  $K(-3, 1)$ ,  $L(2, 1)$ ,  $M(2, 4)$ , and  $N(-3, 4)$ . The parallelogram is translated by the rule  $(x, y) \rightarrow (x+4, y-6)$ . List the coordinates of  $K'L'M'N'$ .

→ original (pre-image)  
 → new (image)  
 → Add 4 to every  $x$ -value  
 → Subtract 6 from every  $y$ -value

$K(-3+4=1, 1-6=-5)$   
 $L(2+4=6, 1-6=-5)$

$M(2+4=6, 4-6=-2)$   
 $N(-3+4=1, 4-6=-2)$

$K'(1, -5)$   
 $L'(6, -5)$   
 $M'(6, -2)$   
 $N'(1, -2)$

18. Point  $B(-5, 3)$  is translated to  $B'(2, -2)$ . Write the algebraic rule for the translation.

$B(-5, 3)$   
 $B'(2, -2)$   
 What's the difference?

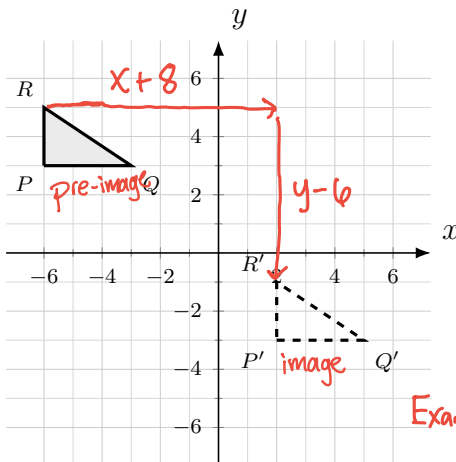
\* image coordinates — pre-image coordinates  
 $x: 2 - (-5) = +7$   
 $y: -2 - 3 = -5$

Algebraic Rule:  
 $(x, y) \rightarrow (x+7, y-5)$

19. A figure is transformed by each rule below. Determine whether each rule preserves or does not preserve congruence. Place a check in the correct column for each row.

| Remember #4 → looking for translations, rotations, reflections for (✓) & just dilations for (x) | Preserves Congruence ✓              | Does Not Preserve Congruence ✗      |
|-------------------------------------------------------------------------------------------------|-------------------------------------|-------------------------------------|
| $(x, y) \rightarrow (x+3, y-2)$ translation (+/- symbols)                                       | <input checked="" type="checkbox"/> | <input type="checkbox"/>            |
| $(x, y) \rightarrow (2x, 2y)$ *multiplication → dilation                                        | <input type="checkbox"/>            | <input checked="" type="checkbox"/> |
| $(x, y) \rightarrow (x-5, y+1)$ translation → All translations preserve congruence.             | <input checked="" type="checkbox"/> | <input type="checkbox"/>            |

20. Triangle  $PQR$  is translated to form triangle  $P'Q'R'$ , as shown.



(a) Write the single algebraic rule that maps triangle  $PQR$  to triangle  $P'Q'R'$ .

$(x, y) \rightarrow (x+8, y-6)$

(b) Write two different translations that, performed one after the other, would move triangle  $PQR$  to the same final position.

\* Any two moves that total  $x$  to +8 &  $y$  to -6

Example 1:  $(x, y) \rightarrow (x+4, y-3); (x, y) \rightarrow (x+4, y-3)$

Example 2:  $(x, y) \rightarrow (x+2, y-1); (x, y) \rightarrow (x+6, y-5)$

21. A figure is translated  $\overbrace{x+p}^{p \text{ units right}}$  and  $\overbrace{y-q}^{q \text{ units down}}$ . A vertex located at  $(a, b)$  maps to which point?
- (A)  $(a + p, b - q)$
- ~~(B)~~  $(a - p, b + q)$
- ~~(C)~~  $(a + q, b - p)$
- ~~(D)~~  $(a + p, b + q)$
- \*Replace  $x$  w/  $a$  &  $y$  w/  $b$
- $(a + p, b - q)$

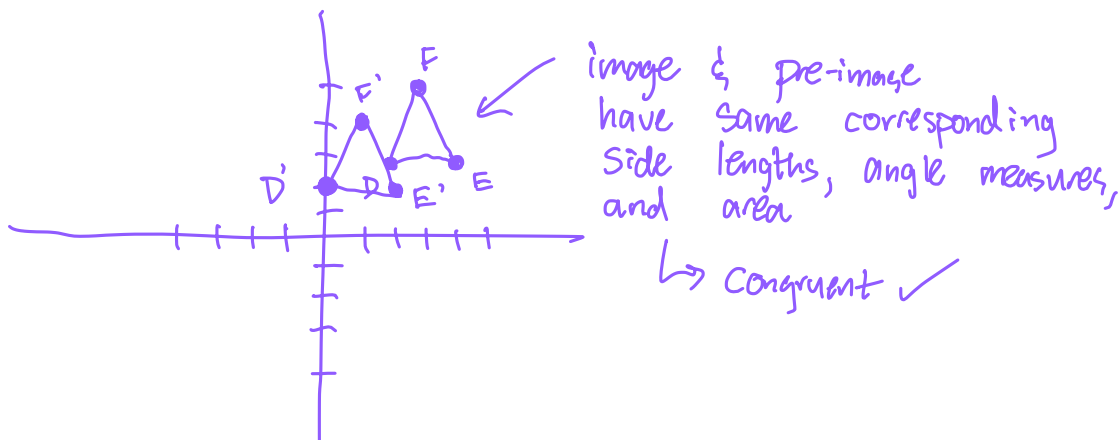
22. Explain why a translation always produces an image that is congruent to the pre-image.

While a translation slides a figure across the coordinate plane, it does so evenly - each vertex of any 2-D figure changes by the same adjustment for the  $x$ -value (left  $\leftrightarrow$  right) &  $y$ -value (down  $\leftrightarrow$  up).

This results in an image that maintains same corresponding side lengths, corresponding angle measures, and areas, and is thus congruent.

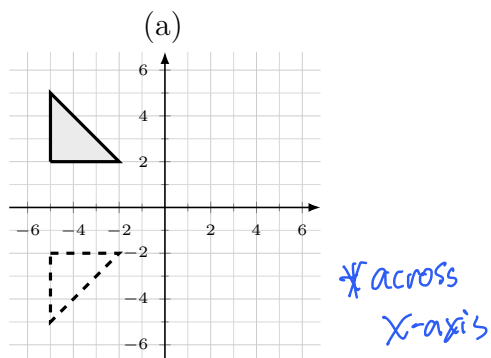
Example:

Triangle DEF is translated 2 units left & 1 unit down. Rule:  $(x, y) \rightarrow (x - 2, y - 1)$



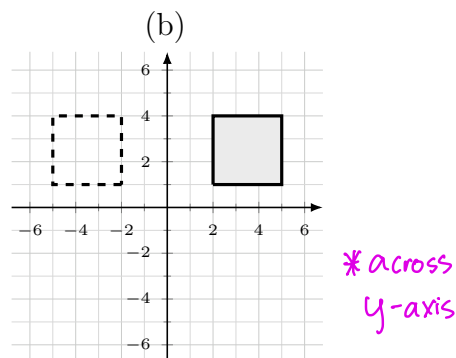
Questions 23-35 cover Artifact 3 (Reflections)

23. For each reflection shown, the shaded figure is the pre-image and the dashed figure is its image. Each figure is reflected across either the  $x$ -axis or the  $y$ -axis. Write the algebraic rule on the line under each graph.



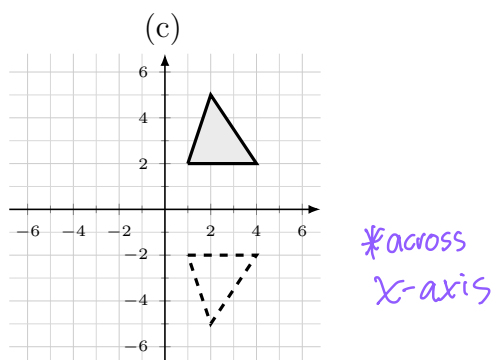
Algebraic Rule:

$$(x, y) \rightarrow (x, -y)$$



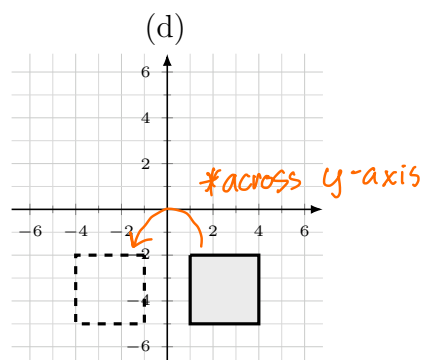
Algebraic Rule:

$$(x, y) \rightarrow (-x, y)$$



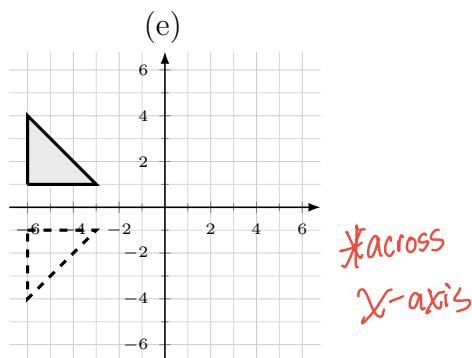
Algebraic Rule:

$$(x, y) \rightarrow (x, -y)$$



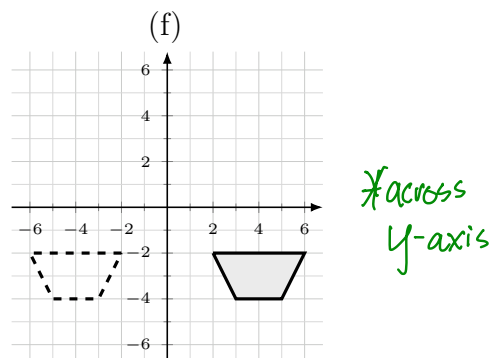
Algebraic Rule:

$$(x, y) \rightarrow (-x, y)$$



Algebraic Rule:

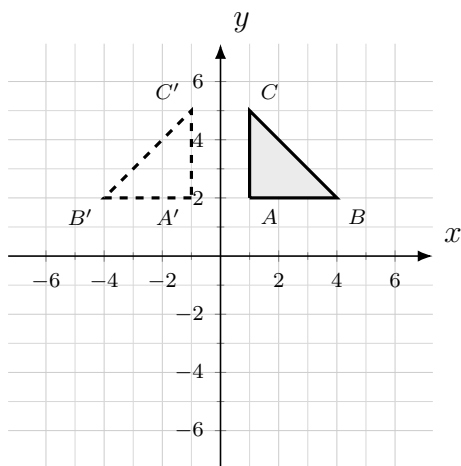
$$(x, y) \rightarrow (x, -y)$$



Algebraic Rule:

$$(x, y) \rightarrow (-x, y)$$

24. Triangle  $ABC$  is reflected to form triangle  $A'B'C'$ , as shown.



- (a) Across which axis is triangle  $ABC$  reflected?

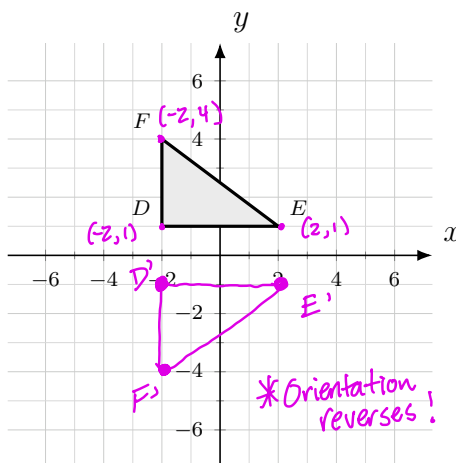
$\triangle ABC$  is reflected across the y-axis to form  $\triangle A'B'C'$ .

- (b) Write the algebraic rule for the reflection.

$$(x, y) \rightarrow (-x, y)$$

25. Triangle  $DEF$  with vertices  $D(-2, 1)$ ,  $E(2, 1)$ , and  $F(-2, 4)$  is reflected across the x-axis. Plot the image  $D'E'F'$  on the grid below, and label the coordinates of each vertex.

$D(-2, 1) \rightarrow D'(-2, -1)$   
 $E(2, 1) \rightarrow E'(2, -1)$   
 $F(-2, 4) \rightarrow F'(-2, -4)$   
 (+ to -)



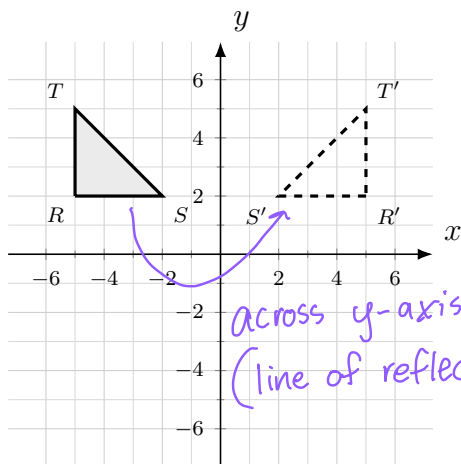
Algebraic Representation:

$$(x, y) \rightarrow (x, -y)$$

\*Keep all x-coordinates the same  
 \*Change the Sign of all y-coordinates

\*Orientation reverses!

26. Triangle  $RST$  is reflected to form triangle  $R'S'T'$ , as shown.



Which rule represents the reflection that maps triangle  $RST$  to triangle  $R'S'T'$ ?

~~(A)~~  $(x, y) \rightarrow (x, -y)$

$\rightarrow$  (B)  $(x, y) \rightarrow (-x, y)$

~~(C)~~  $(x, y) \rightarrow (-x, -y)$

~~(D)~~  $(x, y) \rightarrow (y, x)$

So x must change signs! (y stays the same)

27. Write the algebraic rule for each reflection described. Write your rule in the parentheses.

(a) Quadrilateral  $DEFG$  reflected across the  $x$ -axis

$$(x, y) \rightarrow (x, -y)$$

(b) Triangle  $XYZ$  reflected across the  $y$ -axis

$$(x, y) \rightarrow (-x, y)$$

(c) Pentagon  $JKLMN$  reflected across the  $y$ -axis

$$(x, y) \rightarrow (-x, y)$$

(d) Trapezoid  $TUVW$  reflected across the  $x$ -axis

$$(x, y) \rightarrow (x, -y)$$

28. The coordinates of the vertices of trapezoid  $HJKL$  are  $H(2, 2)$ ,  $J(6, 2)$ ,  $K(5, 5)$ , and  $L(3, 5)$ . The trapezoid is reflected across the  $y$ -axis to create trapezoid  $H'J'K'L'$ . List the coordinates of  $H'J'K'L'$ .

→ Keep  $y$ -values the same  
→ Change signs of all  $x$ -values

$$H'(-2, 2), J'(-6, 2), K'(-5, 5), L'(-3, 5)$$

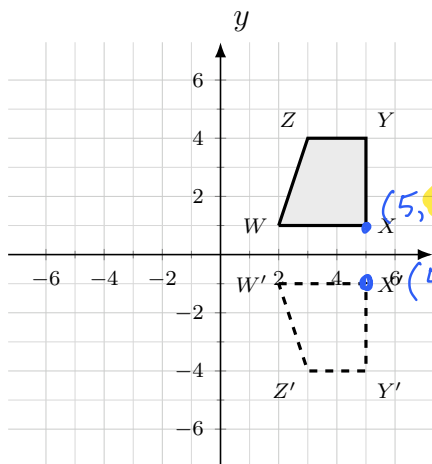
29. Point  $P(-4, 6)$  is reflected across the  $x$ -axis to create point  $P'$ . Then point  $P'$  is reflected across the  $y$ -axis to create point  $P''$ . Give the coordinates of  $P'$  and  $P''$ .

Change  $y$ -value sign (keep  $x$ -value the same)  
Change  $x$ -value sign (keep  $y$  the same)

$$P'(-4, -6)$$

$$P''(4, -6)$$

30. Quadrilateral  $WXYZ$  is reflected to form quadrilateral  $W'X'Y'Z'$ , as shown.



(a) Name the line of reflection.

The  $x$ -axis is the line of reflection.

(b) Write the algebraic rule for the reflection.

$$(x, y) \rightarrow (x, -y)$$

$$(x, y) \rightarrow (x, -y)$$

31. A triangle is graphed on a coordinate grid and then reflected across the  $x$ -axis. If the point  $(x, y)$  is a vertex of the triangle, which ordered pair represents the same vertex of the triangle after the reflection?

~~(A)  $(y, x)$~~

→ (B)  $(x, -y)$

~~(C)  $(-x, y)$~~

~~(D)  $(-x, -y)$~~

\*Change the  $y$  to  $-y$

\*Keep  $x$  the same

\*Note: This is basically describing the algebraic representation (what happens to the coordinate values after a transformation)

32. Hexagon  $ABCDEF$  is reflected across the  $y$ -axis to form hexagon  $A'B'C'D'E'F'$ . Which statement is true?

~~(A) Hexagon  $A'B'C'D'E'F'$  is not congruent to hexagon  $ABCDEF$ .~~

$$(x, y) \rightarrow (-x, y)$$

\*Reflections preserve congruence.

→ (B) The corresponding side lengths of the two hexagons are equal, and the orientation of the vertices is reversed. Yes → reflections reverse orientation!

~~(C) The corresponding angle measures of the two hexagons are equal, and the orientation of the vertices stays the same.~~

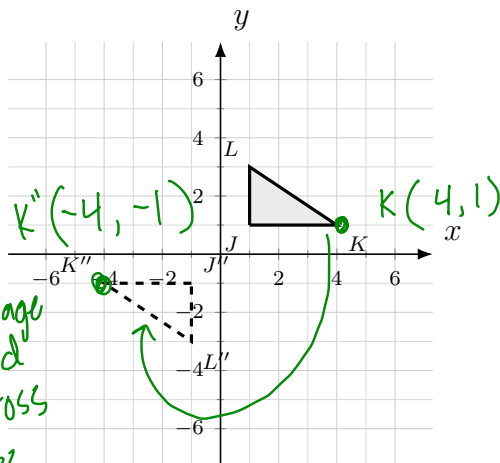
~~(D) The perimeter of hexagon  $A'B'C'D'E'F'$  is greater than the perimeter of hexagon  $ABCDEF$ .~~

33. Determine whether each rule represents a reflection across the  $x$ -axis, a reflection across the  $y$ -axis, or neither. Place a check in the correct column for each row.

|                                                         | Across $x$ -axis                    | Across $y$ -axis                    | Neither                             |
|---------------------------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|
| $(x, y) \rightarrow (-x, y)$                            | <input type="checkbox"/>            | <input checked="" type="checkbox"/> | <input type="checkbox"/>            |
| $(x, y) \rightarrow (x, -y)$                            | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/>            |
| $(x, y) \rightarrow (x + 2, y)$ *This is a translation! | <input type="checkbox"/>            | <input type="checkbox"/>            | <input checked="" type="checkbox"/> |
| $(x, y) \rightarrow (-x, -y)$                           | <input checked="" type="checkbox"/> | <input checked="" type="checkbox"/> | <input type="checkbox"/>            |

→ Reflected across both the  $x$ - and  $y$ -axis!

34. Triangle  $JKL$  is reflected across the  $x$ -axis to form triangle  $J'K'L'$ , and then triangle  $J'K'L'$  is reflected across the  $y$ -axis to form triangle  $J''K''L''$ . The original triangle and the final image are shown.



Just like the description above, this pre-image was reflected twice across both axes, so both coordinates have sign changes.

- (a) Write the single rule that maps triangle  $JKL$  to triangle  $J''K''L''$ .

$$(x, y) \rightarrow (-x, -y)$$

- (b) A reflection across the  $x$ -axis followed by a reflection across the  $y$ -axis is equivalent to what single transformation? Explain how the graph shows this.

This transformation could be considered a rotation, as orientation hasn't reversed, but the image is turned about the origin in the opposite direction.



35. Explain why a reflection across the  $x$ -axis or  $y$ -axis always produces an image that is congruent to the pre-image but with the **reverse** orientation. Use the words corresponding sides, corresponding angles, and orientation in your response.

Reflecting a figure across either axis does not enlarge a shape, as it basically resembles a "flip" across that line of reflection, which reverses the order of the vertices (counting in clockwise or counterclockwise order).

Even though orientation reverses, corresponding sides & angles remain equal in measure, which affirms that the image & pre-image are congruent.

## Questions 36-48 cover Artifact 4 (Rotations)

36. Complete the rotation reference table. Every rotation is about the origin. Write each algebraic rule in the parentheses.

| Rotation about the origin                                     | Algebraic Rule                |
|---------------------------------------------------------------|-------------------------------|
| 90° clockwise (or 270° counterclockwise)                      | $(x, y) \rightarrow (y, -x)$  |
| 90° counterclockwise (or 270° clockwise)                      | $(x, y) \rightarrow (-y, x)$  |
| 180° (clockwise or counterclockwise) *same result, either way | $(x, y) \rightarrow (-x, -y)$ |
| 270° clockwise (or 90° counterclockwise)                      | $(x, y) \rightarrow (-y, x)$  |
| 270° counterclockwise (or 90° clockwise)                      | $(x, y) \rightarrow (y, -x)$  |
| 360° *same result, either way                                 | $(x, y) \rightarrow (x, y)$   |

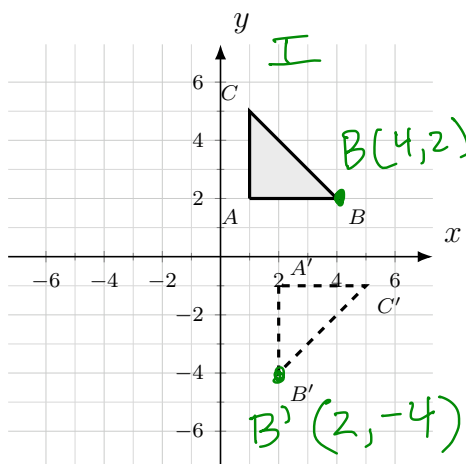
37. Triangle  $MNP$  is rotated 90° clockwise about the origin to create triangle  $M'N'P'$ . Which rule describes this transformation?

- ~~(A)~~  $(x, y) \rightarrow (-y, x)$   
 → (B)  $(x, y) \rightarrow (y, -x)$   
~~(C)~~  $(x, y) \rightarrow (-x, -y)$   
~~(D)~~  $(x, y) \rightarrow (x, -y)$

38. Quadrilateral  $ABCD$  is rotated 180° about the origin to create quadrilateral  $A'B'C'D'$ . Which rule describes this transformation?

- ~~(A)~~  $(x, y) \rightarrow (x, -y)$   
~~(B)~~  $(x, y) \rightarrow (-x, y)$   
 → (C)  $(x, y) \rightarrow (-x, -y)$   
~~(D)~~  $(x, y) \rightarrow (y, x)$

39. Triangle  $ABC$  is rotated about the origin to form triangle  $A'B'C'$ , as shown.



Quadrant I to Quadrant IV

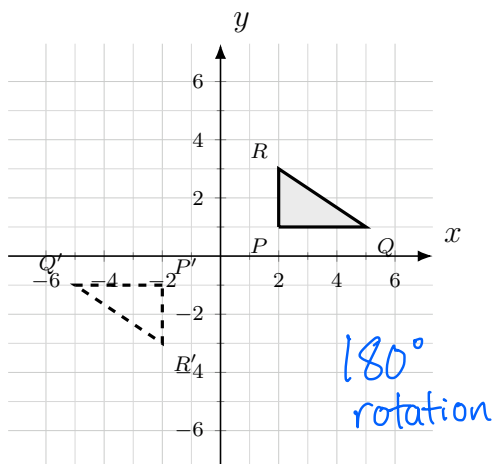
- (a) Describe the rotation (how many degrees, and clockwise or counterclockwise).

- (b) Write the algebraic rule for the rotation.

$$(x, y) \rightarrow (y, -x)$$

→  $(x, y) \rightarrow (y, -x)$   
 90° clockwise  
 ↳ y becomes x (no sign change)  
 ↳ x becomes y (with sign change)

40. Triangle  $PQR$  is rotated about the origin to form triangle  $P'Q'R'$ , as shown.



Which rule represents the rotation that maps triangle  $PQR$  to triangle  $P'Q'R'$ ?

- (A)  $(x, y) \rightarrow (-x, -y)$   
 (B)  $(x, y) \rightarrow (x, -y)$   
 (C)  $(x, y) \rightarrow (-x, y)$   
 (D)  $(x, y) \rightarrow (y, -x)$

41. Write the algebraic rule for each rotation described. Every rotation is about the origin. Write your rule in the parentheses.

- (a)  $90^\circ$  clockwise ( $270^\circ$  counterclockwise)

$$(x, y) \rightarrow (y, -x)$$

- (b)  $90^\circ$  counterclockwise ( $270^\circ$  clockwise)

$$(x, y) \rightarrow (-y, x)$$

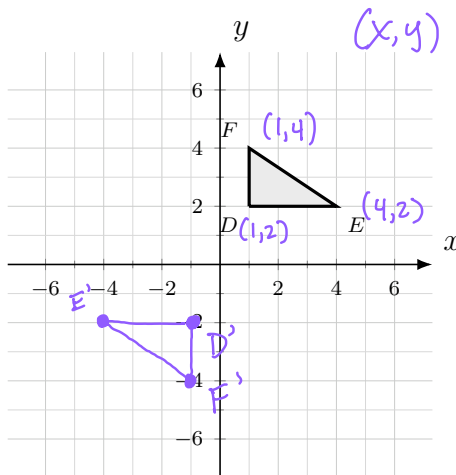
- (c)  $180^\circ$

$$(x, y) \rightarrow (-x, -y)$$

- (d)  $270^\circ$  clockwise ( $90^\circ$  counterclockwise)

$$(x, y) \rightarrow (-y, x)$$

42. Triangle  $DEF$  with vertices  $D(1, 2)$ ,  $E(4, 2)$ , and  $F(1, 4)$  is rotated  $180^\circ$  about the origin. Plot the image  $D'E'F'$  on the grid below, and label the coordinates of each vertex.



$$D(1, 2) \rightarrow D'(-1, -2)$$

$$E(4, 2) \rightarrow E'(-4, -2)$$

$$F(1, 4) \rightarrow F'(-1, -4)$$

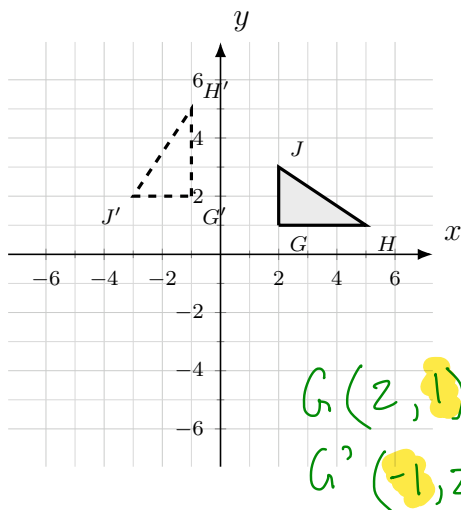
43. Determine whether each rule represents a translation, a reflection, or a rotation. Place a check in the correct column for each row.

|                                                                               | Translation                         | Reflection                          | Rotation                            |
|-------------------------------------------------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|
| $(x, y) \rightarrow (-x, -y)$ 180° rotation                                   | <input type="checkbox"/>            | <input type="checkbox"/>            | <input checked="" type="checkbox"/> |
| $(x, y) \rightarrow (x, -y)$ Reflection across the $x$ -axis                  | <input type="checkbox"/>            | <input checked="" type="checkbox"/> | <input type="checkbox"/>            |
| $(x, y) \rightarrow (x - 3, y + 2)$ Translation (+/-)                         | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/>            |
| $(x, y) \rightarrow (y, -x)$ Rotation: 90° clockwise or 270° counterclockwise | <input type="checkbox"/>            | <input type="checkbox"/>            | <input checked="" type="checkbox"/> |

44. A figure is transformed using the rule  $(x, y) \rightarrow (-x, -y)$ . Which of the following best describes this transformation?

- ~~(A)~~ A reflection across the  $x$ -axis  
~~(B)~~ A reflection across the  $y$ -axis  
 → (C) A 180° rotation about the origin  
~~(D)~~ A 90° clockwise rotation about the origin

45. Triangle  $GHJ$  is rotated about the origin to form triangle  $G'H'J'$ , as shown.



- (a) Describe the rotation (how many degrees, and clockwise or counterclockwise).

90° counterclockwise / 270° clockwise

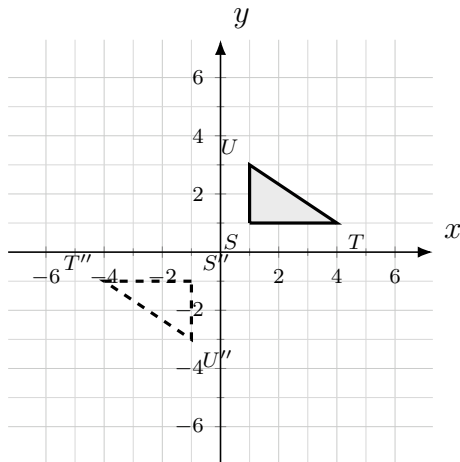
- (b) Write the algebraic rule for the rotation.

$$(x, y) \rightarrow (-y, x)$$

46. Pentagon  $JKLMN$  is rotated 90° counterclockwise about the origin to form pentagon  $J'K'L'M'N'$ . Which statement is true?

- ~~(A)~~ Pentagon  $J'K'L'M'N'$  is not congruent to pentagon  $JKLMN$ .  
 → (B) The corresponding angle measures are equal, and the orientation of the vertices is preserved.  
~~(C)~~ The corresponding side lengths are equal, but the orientation of the vertices is reversed.  
~~(D)~~ The area of pentagon  $J'K'L'M'N'$  is one-fourth the area of pentagon  $JKLMN$ .

47. Triangle  $STU$  is rotated  $90^\circ$  clockwise about the origin to form triangle  $S'T'U'$ , and then triangle  $S'T'U'$  is rotated  $90^\circ$  clockwise about the origin to form triangle  $S''T''U''$ . The original triangle and the final image are shown.



(a) Write the single rule that maps triangle  $STU$  to triangle  $S''T''U''$ .  *$180^\circ$  rotation*

$$(x, y) \rightarrow (-x, -y)$$

(b) Two  $90^\circ$  clockwise rotations about the origin are equivalent to what single rotation? Explain how the graph shows this.

*$180^\circ$  rotation — 2 quarter turns visits 2 quadrants (halfway across the coordinate grid).*

48. A reflection reverses the orientation of a figure, but a rotation preserves it. Explain why a rotation about the origin keeps both the same size and the same orientation as the pre-image. Use the words corresponding sides, corresponding angles, and orientation in your response.

*Rotations, even though they turn the figure, the order of the vertices never change. This is because there is no "flip" around itself resulting in a mirror image, like reflections. However, the image is still congruent to the original pre-image, as corresponding sides and angles are equal in measure.*