Artifact 15 – Writing Linear Equations

Driving Question: When a relationship does not start at zero, how do you find its equation from words, a table, a graph, or two values?

Example 1

A storage unit rents for 65 dollars a month. A customer also pays a 20-dollar move-in fee on the first day. Write an equation for y, the total paid after x months.

Slope/m

one-time/beginning fee → y-intercept

Rate of change, m, and what it means

$m = 65$: each month adds \$65

Initial value, b, and what it means

$b = 20$ → paid at $x = 0$ months (move-in fee)

y-intercept

Equation

$y = 65x + 20$

Total paid after 8 months

$65(8) + 20 = \$540$

Proportional? Justify.

No – we have a non-zero y-intercept

(Equation is not $y = kx$ but $y = mx + b$)

$\frac{y}{x}$ not the same

Example 2

Store B from your board: 30 dollars a week, plus a 55-dollar delivery fee on day one.

Slope/m → $30x$

y-intercept/b → $+55$

Weeks	Total paid
0	55
4	$4 \cdot 30 + 55 = 175$
10	$10 \cdot 30 + 55 = 355$
26	$26 \cdot 30 + 55 = 835$

Rate of change, m, and initial value, b

$m = \$30/\text{week}$

$b = 55$ (initial fee paid at Week 0)

Equation

$y = 30x + 55$

Does $y = 30x$ fit store B? Use a row to decide.

No – $355 \neq 10 \cdot 30$



Where store B's line meets the y-axis, and how it compares with store A's line

(0, 55) → y-intercept → same slope, but this line sits higher (higher starting point)

Example 3

A music school charges a registration fee plus the same price for each lesson. One family had paid 205 dollars in all after 5 lessons and 422 dollars in all after 12 lessons. Write an equation for y, the total paid after x lessons.

slope

Rate of change, m. Show the work.

x	y
5 (x_1)	205 (y_1)
12 (x_2)	422 (y_2)

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{422 - 205}{12 - 5} = 31$

Initial value, b. Show the work.

$y = 31x + b$

$205 = 31 \cdot 5 + b$

$205 = 155 + b$

$-155 \quad -155$

$50 = b$

Equation

$y = 31x + 50$

Check it with the value you did not use

$422 = 31(12) + 50$

$422 = 372 + 50$

$422 = 422$ ✓

Notes to Forgetful Self

Given	Where m comes from	Where b comes from
words	the amount added (or taken away) each time	the amount at the start (initial/beginning)
a table	$\frac{y_2 - y_1}{x_2 - x_1}$ ← $\begin{array}{\|c\|c\|} x & y \\ x_1 & y_1 \\ x_2 & y_2 \end{array}$	the row where $x = 0$ → y-intercept: $\begin{array}{\|c\|c\|} x & y \\ -1 & 2 \\ 0 & 4 \\ 2 & 8 \end{array}$
a graph	Pick 2 points	where the line crosses the y-axis
two values	Same procedure (x_1, y_1) (x_2, y_2)	Count back from one value

y-intercept

A proportional relationship $y = kx$, written as $y = mx + b$:

y-intercept → $b = 0$

$m = k$ (line goes through the origin)