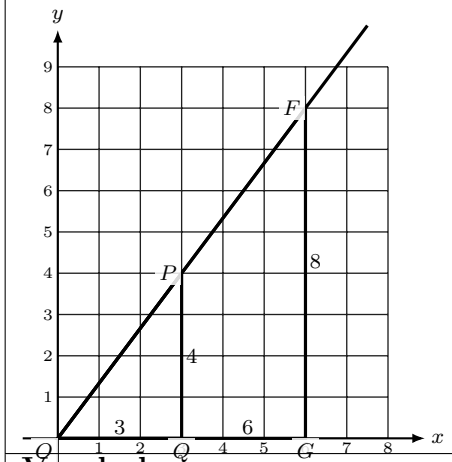
Artifact 11 – Slope and Similar Triangles

Driving Question: Pick any two points on a line. What do their right triangles have in common, and what does that tell you about the line?

Example 1

At one moment a 4-foot sign casts a 3-foot shadow and an 8-foot hoop casts a 6-foot shadow. Every shadow's tip is at the origin; the sun's ray runs through the top of each object.



Right angles at Q and G. The ray makes the same angle at O in both. Two equal angles: triangle OPQ and triangle OFG are **similar right triangles**.

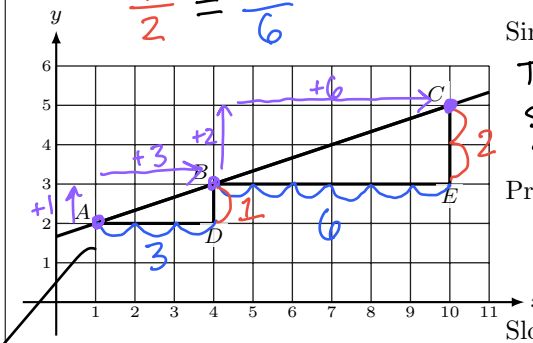
Vertical leg over horizontal leg:

$$\frac{PQ}{OQ} = \frac{4}{3} \quad \text{and} \quad \frac{FG}{OG} = \frac{8}{6} = \frac{4}{3}$$

The same number from either triangle. That number is the **slope** of the ray: $\frac{4}{3}$.

Example 2

Points $A(1,2)$, $B(4,3)$, and $C(10,5)$ lie on the line. Triangle ABD and triangle BCE have their legs along the grid lines and share no corner.



$\frac{1}{2} = \frac{3}{6}$

Similar? Why?

The ratios of corresponding side lengths are the same. $\frac{1}{2} = \frac{3}{6}$ ✓

Proportion, in side names:

$$\frac{BD}{CE} = \frac{AD}{BE}$$

Slope of the line:

$$\frac{\text{Change in } y}{\text{Change in } x} = \frac{1}{3} = \frac{2}{6} \text{ (also equals } \tfrac{1}{3}\text{)}$$

Vocabulary

slope – the steepness of a line: the change in y divided by the change in x between any two points on the line, $\frac{y_2 - y_1}{x_2 - x_1}$

change in x, change in y – the horizontal and vertical differences between two points: $x_2 - x_1$ and $y_2 - y_1$

similar right triangles – right triangles with the same angle measures; any two right triangles drawn on the same line with legs along the grid are similar, so vertical leg ÷ horizontal leg is the same for both

$A(1,2)$ & $B(4,3)$

x_1, y_1 $\quad$ x_2, y_2

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-2}{4-1} = \boxed{\frac{1}{3}}$$

Notes to Forgetful Self

* Any two triangles on the same line are similar (side lengths will always be proportional).

* You can take the vertical (y-change) distance divided by the horizontal (x-change) distance between any two points on the same line to find its slope.